A Gain Scheduled Robust Regulator for Torque Ripple Elimination of AC Permanent Magnet Motor Systems¹

Wai-Chuen Gan and Li Qiu Department of Electrical an Electronic Engineering The Hong Kong University of Science and Technology Clear Water Bay, Kowloon, Hong Kong {eewcgan,eeqiu}@ee.ust.hk

Abstract

This paper addresses the problem of torque and velocity ripple elimination in AC permanent magnet (PM) motor control systems. A gain scheduled (GS) robust two degree of freedom (2DOF) speed regulator based on the internal model principle (IMP) and the pole-zero placement is developed to eliminate the torque and velocity ripples and achieve a desirable tracking response.



1 Introduction

Precision speed control systems such as feed control of machine tools in the manufacturing industry are crucial in numerous industrial applications [1]. AC permanent magnet (PM) motors are attractive candidates for high performance industrial control applications as the maintenance of AC PM motors is minimal due to the brushless rotor construction. However, the torque ripple generation in AC PM motor systems limits the applications of AC PM motors in high performance speed and position control systems. In general, the disturbance torque ripples of AC PM motor control systems are composed of cogging torque, reluctance torque, mutual torque and the DC current offset torque that is caused by the DC offsets of the current sensors in the motor driver and the digital-to-analog converters in the motion controller. For optimally designed AC PM motors, cogging, reluctance and mutual torque ripples can be neglected [2, 3]. The torque ripples due to current offsets are dominant among the above four types of ripples in a typical AC PM motor control system as the offsets from the current sensors and digital-to-analog converters are difficult to eliminate.

In a practical high performance AC PM motor control system, the basic components consist of a motion controller, a current tracking amplifier, a feedback encoder and an AC PM motor as shown in Fig. 1. DC offsets are always present at the motor terminals due to

 $^1\mathrm{This}$ work is supported by Hong Kong Research Grants Council under grant HKUST6046/00E

Figure 1: An AC PM motor control system.

the digital-to-analog converter offsets of the motion controller and the current sensor offsets of the current tracking amplifier. These current offsets generate sinusoidal torque disturbances and hence produce velocity ripples at the system output. In the past, the way to eliminate these torque ripples was to purchase high-grade and expensive current sensors and digital-to-analog converters to minimize the DC offset values respectively; the total cost of an automation machine is thus much increased. To replace the above inefficient and high-cost solutions, the development of a novel and cost-effective speed control algorithm to eliminate the torque ripples caused by DC current offsets is discussed in this paper.

The AC PM motor used in our motor control system is assumed to be well designed so that the cogging torque, reluctance torque and mutual torque can be neglected, and this assumption is often valid in high performance applications [2]. On the other hand, we focus on the elimination of the torque ripples caused by DC current offsets. The internal model principle (IMP) is applied in the motor controller design to eliminate the torque ripples without estimating the amplitude and the phase values of the sinusoidal disturbance. A gain scheduled (GS) robust two degree of freedom (2DOF) speed regulator based on the IMP and the pole-zero placement algorithm is designed so as to achieve a desirable and velocity ripple-free output response for a time-varying step reference input.

2 Vector Control of AC PM motors and Disturbance Modeling

In this section the vector control of AC PM motors is reviewed and the modeling of the torque ripple disturbance caused by the DC current offsets is discussed. With a current controlled AC PM motor, the d-q frame current, $i_d(t)$ and $i_q(t)$ are the system inputs. Vector control technique suggests to set $i_d(t) = 0$. This converts the nonlinear AC PM motor system into a linear system [5]:

$$\tau_e(t) = \frac{3}{2} \frac{P}{2} \lambda_m i_q(t)$$

$$\tau_e(t) - \tau_l(t) = J_m \frac{d\omega(t)}{dt} + B_m \omega(t)$$

where $\tau_e(t)$ is the generated torque, $\tau_l(t)$ is the load torque, J_m is the moment of inertia, B_m is the friction constant, λ_m is the constant magnetic flux and P is the number of poles.

When the motor current amplifier is connected to the power source and the two current reference commands from the motion controller are kept at zero, a DC offset current induced by the current sensor offsets and the digital-to-analog converter offsets may be present in one or both of the closed loop controlled phases and, thus, also in the third one [3]. Let I_a , I_b be the two DC current offsets present at the motor terminals due to the digital-to-analog converter offsets of the motion controller and the current sensor offsets of the current amplifier. Then the torque ripples, $\tau_{off}(t)$, for AC PM motors with $L_d = L_q$ is given by [4]

$$\begin{split} \tau_{off}(t) &= -\frac{P}{2} \lambda_m I_a \left[-\frac{3}{2} \sin(\theta_e(t)) + \frac{\sqrt{3}}{2} \cos(\theta_e(t)) \right] + \\ &- \frac{P}{2} \lambda_m I_b \left[\sqrt{3} \cos(\theta_e(t)) \right] \end{split}$$

where $\theta_e(t)$ is the rotor electrical angle. When the output mechanical speed tracks closely with the reference input at the steady state, it follows that the relationship $\omega_d(t) = \omega_e(t) \approx \frac{P}{2} \omega_r(t)$ can be assumed where $\omega_d(t)$ is defined as the disturbance frequency. The disturbance $\tau_{off}(t)$ can then be further simplified and approximated by a sinusoidal function:

$$\tau_{off}(t) = A_d \cos(\omega_d(t)t - \phi_d) \tag{1}$$

where A_d is the magnitude of the disturbance while ϕ_d is the phase of the disturbance. The disturbance frequency, $\omega_d(t)$, is a slowly time-varying function in comparison to the cosine function.

In summary, after employing the vector control and the formulation of the sinusoidal disturbance, the model of a vector controlled AC PM motor with the torque ripple disturbance is given by Fig. 2. Here, $u(t) = i_q(t)$ is the input current, $y(t) = \omega(t)$ is the output mechanical speed, $K_t = \frac{3}{2} \frac{P}{2} \lambda_m$ is the equivalent torque constant, $d(t) = \frac{\tau_{\rm eff}(t) + \tau_i(t)}{K_i}$ where $\tau_l(t)$ is the load torque which can be considered as an unknown constant disturbance, $\tau_{\rm off}(t)$ is the torque disturbance due to DC current offsets, and can be approximated by a sinusoidal function with a known frequency $\omega_d(t)$ and an unknown magnitude and phase. Our goal is to design a speed controller so that the output speed tracks a constant reference or a time-varying step reference and rejects the disturbance $\tau_{\rm off}(t)$ and $\tau_l(t)$. Such a controller is required to be robust, i.e. to perform the tracking and disturbance rejection even when the system parameters vary slightly, to have a good transient response, and to have low complexity, i.e. to have an order as low as possible.

To eliminate completely and robustly the sinusoidal disturbance with the known frequency, the use of the IMP is necessary. The IMP calls the use of the modes of the disturbance in the controller as internal modes of the feedback loop. The estimation of the amplitude and the phase values of the disturbance is not necessary in the elimination of the sinusoidal disturbance.

3 Controller Design

The problem of accomplishing robust tracking and disturbance rejection is called the robust regulator problem. The key idea to solve a robust regulator problem is, based on the IMP, to have the controller to include the modes of the reference and disturbance. We also propose to use a 2DOF controller structure to achieve better transient responses and simpler designs. One of its advantages, in comparison with the usual one degree of freedom or unity feedback structure, is that the tracking performance and the disturbance rejection performance can be designed with different considerations.

In this section we first investigate the design of a linear time invariant (LTI) robust 2DOF controller for a constant speed reference. Then a GS robust 2DOF controller for a slowly time-varying speed step reference is designed by modifying the LTI controller.

3.1 Controller Design for a Constant Reference

In this section the input reference is assumed to be a constant value ω_r so that $\omega_d = \frac{P}{2}\omega_r$. Robust 2DOF regulators were discussed in [6], in which the disturbance



Figure 2: The system model with disturbance.

and reference are assumed to have the same modes. In the following we assume that they may have different modes. The main purpose of this section is to develop a simple pole-zero placement design method for robust 2DOF regulators.

The 2DOF regulator structure employed in our analysis is shown in Fig. 3 with n denotes the sensor noise. The detail design procedure for the robust 2DOF regulator using pole-zero placement technique is given in [4]. For our PM motor control system, in reference to Fig. 3, we have $a(s) = s + \frac{B_m}{J_m}$ and $b(s) = \frac{K_1}{J_m}$. Since the reference r(t) is a step reference, it follows that $m_r(s) = s$. Since the disturbance d(t) contains a sinusoidal function of frequency $\omega_d = \frac{P}{2}\omega_r$ and a constant function, it follows that $m_d(s) = s(s^2 + \omega_d^2)$. Therefore, $m(s) = s(s^2 + \omega_d^2)$. It follows that m(s)a(s) and b(s) are coprime and a solution to the robust regulator problem based on the IMP exists. Since $n_a = 1$, we can choose $n_g = 0$. This leads to a controller of order equal to n_m , which is the lowest possible to achieve robust regulator.

$$k(s) = m(s) = s(s^2 + \omega_d^2)$$
 (2)

and h(s) and q(s) have the following forms:

$$\begin{aligned} h(s) &= h_0 s^3 + h_1 s^2 + h_2 s + h_3 \\ q(s) &= q_0 s^3 + q_1 s^2 + q_2 s + q_3. \end{aligned}$$

Choose the closed loop poles α_1 , α_2 and α_3 according to the disturbance rejection specification and the remaining closed loop pole α_4 according to the transient tracking response specification so that the closed loop characteristic polynomial is

$$\delta(s) = (s + \alpha_1)(s + \alpha_2)(s + \alpha_3)(s + \alpha_4) = s^4 + \delta_1 s^3 + \delta_2 s^2 + \delta_3 s + \delta_4$$

Then by equating the coefficients of both sides of

$$\delta(s) = k(s)a(s) + b(s)h(s)$$

we can obtain

$$h_0 = \frac{J_m}{K_t} \left(\delta_1 - \frac{B_m}{J_m} \right) \tag{3}$$

$$h_1 = \frac{J_m}{K_t} \left(\delta_2 - \omega_d^2 \right) \tag{4}$$



Figure 3: The 2DOF regulator structure.

$$h_2 = \frac{J_m}{K_t} \left(\delta_3 - \omega_d^2 \frac{B_m}{J_m} \right) \tag{5}$$

$$h_3 = \frac{J_m}{K_t} \delta_4. \tag{6}$$

Finally, as $m_r(s) = s$, we can arbitrarily assign the roots of q(s). Here we choose the three roots of q(s) to be exactly the same as three roots of $\delta(s)$ subject to the constraint $q_3 = h_3$,

$$q(s) = \frac{h_3}{\alpha_1 \alpha_2 \alpha_3} (s + \alpha_1)(s + \alpha_2)(s + \alpha_3) \tag{7}$$

where

$$q_0 = \frac{h_3}{\alpha_1 \alpha_2 \alpha_3}$$

$$q_1 = \frac{h_3(\alpha_1 + \alpha_2 + \alpha_3)}{\alpha_1 \alpha_2 \alpha_3}$$

$$q_2 = \frac{h_3(\alpha_1 \alpha_2 + \alpha_2 \alpha_3 + \alpha_3 \alpha_1)}{\alpha_1 \alpha_2 \alpha_3}$$

$$q_3 = h_3.$$

In this way, the system from r to y is turned to a first order system with a pole determined by the remaining root of $\delta(s)$ and the transfer function is given by

$$\frac{Y(s)}{R(s)} = \frac{\alpha_4}{s + \alpha_4}$$

3.2 Controller Design for a Time-Varying Step Reference

The robust 2DOF regulator described in the previous section is designed for a constant speed reference so that the tracking error is zero at the steady state. However, there are many industrial applications that require a varying speed operation. In this case, the sinusoidal disturbance has a varying frequency. In order to achieve zero steady state error, we need to include an internal mode which varies with the disturbance frequency; other parameters of the controller in general also need to be changed with time to ensure that the closed loop system, which is time-varying, is internally and externally stable. In this section the development of a GS robust 2DOF regulator for a time-varying step reference to eliminate the torque and velocity ripples of AC PM motors, is discussed.

The idea behind the GS robust 2DOF pole-zero placement regulator is to replace the disturbance frequency ω_d in (2) and (3)-(6) by $\omega_d(t) = \frac{P}{2}\omega_r(t)$ as follows:

$$k_2(t) = \omega_d^2(t) \tag{8}$$

$$h_0(t) = \frac{J_m}{K_t} \left(\delta_1 - \frac{J_m}{J_m} \right) \tag{9}$$

$$h_1(t) = \frac{J_m}{K_t} \left(\delta_2 - \omega_d^2(t) \right) \tag{10}$$

286

$$h_2(t) = \frac{J_m}{K_t} \left(\delta_3 - \omega_d^2(t) \frac{B_m}{J_m} \right) \tag{11}$$

$$h_3(t) = \frac{J_m}{K_t} \delta_4 \tag{12}$$

$$f_0(t) = \frac{J_m}{K_t} \left(\delta_1 - \frac{B_m}{J_m} \right) - q_0 \tag{13}$$

$$f_1(t) = \frac{J_m}{K_t} \left(\delta_2 - \omega_d^2(t) \right) - q_1$$
 (14)

$$f_2(t) = \frac{J_m}{K_t} \left(\delta_3 - \omega_d^2(t) \frac{B_m}{J_m} \right) - q_2 \qquad (15)$$

where $\omega_d(t)$ is now the time-varying frequency of the disturbance. The coefficients of q(s) are constants as in (7). Fig. 4 shows the block diagram of the GS robust 2DOF speed regulator. Let $\hat{x}(t) = [x_1(t) \ x_2(t) \ x_3(t)]'$ and $\hat{u}(t) = [r(t) \ y(t) + n(t)]'$, the regulator can be implemented using the following observer canonical realization:

$$\dot{\hat{x}}(t) = A_K(t)\hat{x}(t) + \begin{bmatrix} B_{K1}(t) & B_{K2}(t) \end{bmatrix} \hat{u}(t) u(t) = C_K(t)\hat{x}(t) + \begin{bmatrix} D_{K1}(t) & D_{K2}(t) \end{bmatrix} \hat{u}(t)$$

where

$$\begin{bmatrix} A_{K}(t) & B_{K1}(t) & B_{K2}(t) \\ \hline C_{K}(t) & D_{K1}(t) & D_{K2}(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & q_{1} & -h_{1}(t) \\ -\omega_{d}^{2}(t) & 0 & 1 & q_{2} - q_{0}\omega_{d}^{2}(t) & -h_{2}(t) + h_{0}(t)\omega_{d}^{2}(t) \\ \hline 0 & 0 & 0 & q_{3} & -h_{3}(t) \\ \hline 1 & 0 & 0 & q_{0} & -h_{0}(t) \end{bmatrix}$$

Since we need to deal with time-varying controller parameters, the transfer function argument is no longer valid and instead, the state space theory has to be invoked [8]. In the following, the stability of the time-varying closed loop system under the control by the proposed GS robust speed regulator is studied. The linearized AC PM motor system plant can be represented by the following state space equations:

$$\dot{x}_4(t) = -a_1 x_4(t) + b_1(u(t) + d(t))$$

 $y(t) = x_4(t)$

where $a_1 = B_m/J_m$, $b_1 = K_t/J_m$, $x_4(t)$ is the state variable, u(t) is the control input, d(t) is the disturbance input and y(t) is the control output. Let $\tilde{x}(t) =$



Figure 4: The GS robust 2DOF speed regulator.

 $[x_1(t) \ x_2(t) \ x_3(t) \ x_4(t)]', \ \tilde{u}(t) = [r(t) \ d(t) \ n(t)]'$ and $\tilde{y}(t) = [u(t) \ y(t)]'$, the closed loop state space equations can be written as follows in reference to Fig. 3:

$$\dot{\tilde{x}}(t) = A(t)\tilde{x}(t) + B(t)\tilde{u}(t)$$

$$\tilde{y}(t) = C(t)\tilde{x}(t) + D(t)\tilde{u}(t)$$

where the system matrices are defined in (16).

The above system matrices can be transformed into an observer canonical form so as to facilitate the stability analysis by small gain theorem. A transformation matrix P(t) is chosen accordingly to perform the task. Let $z(t) = [z_1(t) \ z_2(t) \ z_3(t) \ z_4(t)]'$ be the new state variables. With the following transformation,

$$\tilde{x}(t) = P(t)z(t) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & -\omega_d^2(t) \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & b_1 \end{bmatrix} z(t),$$

the closed loop system state space equations can be transformed into the following equations,

$$\begin{aligned} \dot{z}(t) &= A_z(t)z(t) + B_z(t)\tilde{u}(t) \\ \tilde{y}(t) &= C_z(t)z(t) + D_z(t)\tilde{u}(t) \end{aligned}$$

where

$$\begin{bmatrix} A_{z}(t) & B_{z}(t) \\ \hline C_{z}(t) & D_{z}(t) \end{bmatrix}$$

$$= \begin{bmatrix} P^{-1}(t)A(t)P(t) - P^{-1}(t)\dot{P}(t) & P^{-1}(t)B(t) \\ \hline C(t)P(t) & D(t) \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 & -\delta_{4} & q_{3} & 0 & -h_{3}(t) \\ 1 & 0 & 0 & -\delta_{3} + (\omega_{d}^{2}(t))' & q_{2} & \omega_{d}^{2}(t) & -h_{2}(t) \\ 0 & 1 & 0 & -\delta_{2} & q_{1} & 0 & -h_{1}(t) \\ 0 & 0 & 1 & -\delta_{1} & q_{0} & 0 & -h_{0}(t) \\ \hline 0 & 0 & 1 & -b_{1}h_{0}(t) & q_{0} & 0 & -h_{0}(t) \\ 0 & 0 & 0 & b_{1} & 0 & 0 & 0 \end{bmatrix}$$

The polynomial $\delta(s) = s^4 + \delta_1 s^3 + \delta_2 s^2 + \delta_3 s + \delta_4$ is the desired closed loop characteristic equation. According to the small gain theorem [7], the associated autonomous system

$$\begin{aligned} \dot{z}(t) &= A_z(t)z(t) = \\ \begin{pmatrix} 0 & 0 & 0 & -\delta_4 \\ 1 & 0 & 0 & -\delta_3 \\ 0 & 1 & 0 & -\delta_2 \\ 0 & 0 & 1 & -\delta_1 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} (\omega_d^2(t))' \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}') z(t) \end{aligned}$$

is internally uniformly exponentially stable if the following condition is satisfied:

$$\begin{split} &|(\omega_{d}^{2}(t))'| \\ < \left\| \begin{bmatrix} 0\\0\\0\\1 \end{bmatrix} \right|' \left(sI - \begin{bmatrix} 0&0&0&-\delta_{4}\\1&0&0&-\delta_{3}\\0&1&0&-\delta_{2}\\0&0&1&-\delta_{1} \end{bmatrix} \right)^{-1} \begin{bmatrix} 0\\1\\0\\0 \end{bmatrix} \right\|_{\infty}^{-1} \\ = \left\| \frac{s}{s^{4} + \delta_{1}s^{3} + \delta_{2}s^{2} + \delta_{3}s + \delta_{4}} \right\|_{\infty}^{-1} := r_{S} \end{split}$$
(17)

$\left[\begin{array}{c c} A(t) & B(t) \\ \hline C(t) & D(t) \end{array}\right] =$	$ \begin{array}{c} 0 \\ -\omega_d^2(t) \\ 0 \\ \underline{b_1} \\ 1 \\ 0 \end{array} $		0 1 0 0 0	$\begin{array}{c} -h_1(t) \\ -h_2(t) + h_0(t) \omega_d^2(t) \\ -h_3(t) \\ -a_1 - b_1 h_0(t) \\ \hline -h_0(t) \\ 1 \end{array}$	$\begin{array}{c} q_1 \\ q_2 - q_0 \omega_d^2(t) \\ q_3 \\ b_1 q_0 \\ q_0 \\ 0 \end{array}$	$ \begin{array}{c} 0 \\ 0 \\ 0 \\ b_1 \\ 0 \\ 0 \end{array} $	$\begin{array}{c} -h_1(t) \\ -h_2(t) + h_0(t) \omega_d^2(t) \\ -h_3(t) \\ -b_1 h_0(t) \\ \hline \\ -h_0(t) \\ 0 \end{array}$	(16)
	0	0	0	1	0	0	0]	

where r_S is defined as the stability radius. Therefore, as long as the input reference does not change too fast, the condition in (17) is satisfied, and the internal stability of the closed loop system is preserved. Since the matrices $B_z(t)$, $C_z(t)$ and $D_z(t)$ associated with the closed loop system are bounded, it follows from [8], the closed loop system is also bounded input bounded output (BIBO) stable if (17) is satisfied.

As $\omega_d(t) = \frac{P}{2}\omega_r(t)$, if the differentiation of the input speed reference $\omega_r(t)'$, or the acceleration profile, is available to the controller, then the stability radius can be further enlarged. By modifying the feedback gain $h_2(t)$ in (11) as:

$$h_{2}(t) = \frac{J_{m}}{K_{t}} \left[\delta_{3} - \omega_{d}^{2}(t) \frac{B_{m}}{J_{m}} - (\omega_{d}^{2}(t)') \right]$$
(18)

Now the closed loop system matrices are given by

$$\begin{bmatrix} A_z(t) & B_z(t) \\ \hline C_z(t) & D_z(t) \end{bmatrix} = \\ \begin{bmatrix} 0 & 0 & 0 & -\delta_4 & q_3 & 0 & -h_3(t) \\ 1 & 0 & 0 & -\delta_3 & q_2 & \omega_d^2(t) & -h_2(t) \\ 0 & 1 & 0 & -\delta_2 & q_1 & 0 & -h_1(t) \\ 0 & 0 & 1 & -\delta_1 & q_0 & 1 & -h_0(t) \\ \hline 0 & 0 & 1 & -b_1h_0(t) & q_0 & 0 & -h_0(t) \\ 0 & 0 & 0 & b_1 & 0 & 0 & 0 \end{bmatrix}$$

As now $A_z(t)$ is a constant matrix and the other system matrices $B_z(t)$, $C_z(t)$ and $D_z(t)$ are bounded, the system is internally stable and BIBO stable.

In summary, the GS robust 2DOF regulators based on the IMP and the pole-zero placement algorithm can be implemented with an infinity stability radius provided that the differentiation of the input speed reference $\omega_r(t)'$ is available for the input of the GS robust 2DOF regulators. In this case, the new gain scheduled terms are given by (8)-(12) except that $h_2(t)$ is given by (18).

4 Simulation and Experimental Results

Simulations are first performed to test the proposed control algorithm. A 50W AC PM motor is used in our simulations and experimental tests. The motor parameters are listed in Table 1. In the simulation, we assume that a -0.08A current offset is present at the current sensor of phase 1 and a 0.05A current offset at the current sensor of phase 2.

The GS robust 2DOF speed regulator with the timevarying gain (8)-(12) is tested with a time-varying step reference. With the addition of the acceleration profile input, the improved GS robust 2DOF speed regulator with the new time-varying gain (18) can be implemented so as to enlarge the stability radius.

The GS robust 2DOF speed regulator is first designed to satisfy a step rise time < 60ms. According to the design procedure listed in Section 3, one of the possible solutions is to choose the four closed loop poles at -40, -50, -60 and -80, and the three closed loop zeros at -50, -60 and -80. Hence the overall system is a first order system with a single pole at -40, the slowest closed loop pole. With this selection, the rise time of the closed loop system is around 54ms< 60ms. The stability of this GS robust 2DOF speed regulator depends on the stability radius r_S and the differentiation of the input reference $|(\frac{P^2}{4}\omega_r^2(t))'|$ in (17). An input profile with the condition $|(\frac{P^2}{4}\omega_r^2(t))'| > r_S = 5.57 \times 10^5$, is tested with the proposed GS robust 2DOF speed regulator.

Without the acceleration profile input, the speed response is shown in the upper section of Fig. 5; the output speed response is not good in general and at t = 3s, as the value of $|(\frac{P^2}{4}\omega_r^2(t))'| > r_S$, the output speed becomes unstable and oscillatory. However, if the improved GS robust 2DOF speed regulator is implemented with the acceleration profile input, a comparatively good speed response is obtained and shown in the lower section of Fig. 5, and the stability of the whole system remains. Smooth transitions can still be maintained during the high speed turning corners at t = 2s and t = 3s. This simulation result shows the stability radius of the overall system can be enlarged with the availability of the acceleration input profile.

Table 1: Motor parameters.

J_m	$0.144 \times 10^{-4} \mathrm{kg} \cdot \mathrm{m}^2$
B_m	$5.416 \times 10^{-4} \mathrm{Nm/rad \cdot s^{-1}}$
λ_m	0.0283Wb
L_d, L_q	11.5mH
P	8
$K_t = \frac{3}{2} \frac{P}{2} \lambda_m$	0.1698Nm/A
Encoder resolution	8000 counts/rev
Rated speed	1000rpm



Figure 5: The simulation output speed responses.

In the practical experiment, another input profile with the maximum value of $|(\frac{p^2}{4}\omega_r^2(t))'| < r_S$ is employed. Fig. 6 depicts the experimental speed response when the improved GS robust 2DOF speed regulator is used; the output speed does not contain any ripple and achieves a desirable tracking response. The time-varying sinusoidal modes inside the regulator can generate an equal but opposite signal to eliminate the torque and velocity ripples cause by the DC current offsets.

The simulation and experimental results shown in this section demonstrated that the improved GS robust 2DOF speed regulator based on the IMP is an effective solution to eliminate the torque ripples in an AC PM motor control system.



Figure 6: The experimental output speed response.

5 Conclusions

In this paper the use of IMP to eliminate the torque and velocity ripples caused by DC current offsets in AC PM control systems is demonstrated to be an novel and effective solution. The model of the torque ripples caused by DC current offsets is developed. Then a GS robust 2DOF speed regulator based on the IMP is designed to eliminate the torque and velocity ripples for a timevarying speed step reference. The stability of this speed regulator depends on the closed loop system poles and the availability of the acceleration input profile. Another improved GS robust 2DOF speed regulator with the addition of the acceleration profile input, is constructed so as to maintain the stability of the closed loop system all the time.

Simulation and experimental results reveal that the improved GS robust 2DOF speed regulator can eliminate the torque and velocity ripples successfully without estimating the amplitude and phase values of the disturbance directly.

References

[1] D. Chen and B. Paden, "Adaptive linearization of hybrid step motors: stability analysis," *IEEE Trans. Automatic Control*, vol. 38, no. 6, pp. 874-887, 1993.

[2] V. Petrovic, R. Ortega, A. M. Stankovic and G. Tadmor, "Design and implementation of an adaptive controller for torque ripple minimization in PM synchronous motors," *IEEE Trans. Power Electronics*, vol. 15, no. 5, pp. 871-880, Sep. 2000.

[3] G. Ferretti, G. Magnani and P. Rocco, "Modeling, identification, and compensation of pulsating torque in permanent magnet AC motors," *IEEE Trans. Industrial Electronics*, vol. 45, no. 6, pp. 912-920, Apr. 1998.

[4] W. C. Gan and L. Qiu, "Robust two degree of freedom regulators for velocity ripple elimination of AC permanent magnet motors," *Proc. of the Ninth IEEE International Conference on Control Applications*, vol. 1, pp. 156-161, Sep. 2000.

[5] D. W. Novotny and T. A. Lipo, Vector Control and Dynamics of AC Drives, Oxford, 1998.

[6] W. A. Wolovich, Automatic Control Systems, Sauders College Publishing, 1994.

[7] C. A. Desoer and M. Vidyasagar, *Feedback Systems:* Input-Output Properties, Academic Press, New York, 1975.

[8] W. J. Rugh, *Linear System Theory*, Prentice Hall, 1996.