Concluding Remark: One should note that if Conditions C3)–C5) are supposed to hold, p and M can be easily identified from the truncated sequence $(R_k)_{k\geq 1}$. In fact, M is the smallest index for which $R_k = 0$ for each $k \geq M+1$. On the other hand, $R_M = H_0 H_M^T$. Therefore, $p = \text{Rank} R_M$. However, one should note that it seems to be difficult to check that C3)–C5) hold.

References

[1] K. Abed Meraim, P. Loubaton, and E. Moulines, "A subspace algorithm for certain blind identification problems," *IEEE Trans. Information Theory*, to appear.

[2] K. Abed Meraim, "Second order blind identification/equalization: Algorithms and statistical performance," Ph.D. dissertation, Ecole Nationale Supérieure des Télécommunications, Paris, July 1995.

[3] I. Fijalkow and P. Loubaton, "Identification of rank one rational spectral densities from noisy observations: A stochastic realization approach," *Syst. Contr. Lett.* vol. 24, pp. 201–205, 1995.

[4] G. D. Forney, "Minimal bases of rational vector spaces, with applications to multivariable linear systems," *SIAM J. Contr.*, vol. 13, no. 3, pp. 493–520, May 1975.

[5] T. Kailath, *Linear Systems*. Englewood Cliffs, NJ: Prentice-Hall, 1980.

[6] S. Kung, T. Kailath, and M. Morf, "A generalized resultant matrix for polynomial matrices," in *Proc. IEEE Conf. Decision Contr.*, 1976.

[7] L. Tong, G. Xu, and T. Kailath, "A new approach to blind identification and equalization of multipath channels," in *Proc. Asilomar Conf.*, 1991, pp. 856–860.

[8] A. van der Veen, S. Talwar, and A. Paulraj, "Blind identification of FIR channels carrying multiple finite alphabet signals," in *Proc. ICASSP*, Detroit, MI, May 1995, pp. 1213–1216.

A Single Sample Path-Based Performance Sensitivity Formula for Markov Chains

Xi-Ren Cao, Xue-Ming Yuan, and Li Qiu

***Abstract*—Using a sample path approach, we derive a new formula for performance sensitivities of discrete-time Markov chains. A distinguished feature of this formula is that the quantities involved can be estimated by analyzing a single sample path of a Markov chain. Thus, the formula provides a new direction for sensitivity analysis and can be viewed as an extension of the perturbation realization theory to problems where infinitesimal perturbation analysis does not work well.**

I. Introduction

In this paper, we derive a new formula for performance sensitivities of homogenous discrete-time Markov chains (we shall simply use the term Markov chain hereafter). We use a sample path approach, i.e., we analyze a sample path to determine the performance change due to a change in the transition probability matrix. The formula shows that the derivative of a performance measure equals the weighted sum of the expected value of a quantity, called a *realization factor*, that measures the average performance change when the Markov chain changes from one state to another. The realization factors can be determined by solving a skew-symmetric Lyapunov equation. Furthermore, these realization factors can be estimated by analyzing a single sample path obtained from simulation or a record of a real system.

Manuscript received August 22, 1995; revised March 5, 1996. This work was supported in part by Hong Kong UPGC under Grant HKUST 599/94E and by the Hongkong Telecom Institute of Information Technology.

X.-R. Cao and L. Qiu are with the The Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong (e-mail: eecao@ee.ust.hk).

X.-M. Yuan is with the The Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong and is on leave from the Chinese Academy of Sciences.

Publisher Item Identifier S 0018-9286(96)08445-0.

The formula provides a new perspective to the sensitivity analysis of Markov chains. It shows that a single sample path contains all the information needed for determining the performance sensitivities in a Markov chain. The approach can be viewed as an extension of the realization theory [1] in infinitesimal perturbation analysis (IPA) [5], [6] to the case where a small change may induce a large change in the sample path. This work was motivated by a recent work of Dai and Ho [4] and can be considered a theoretical justification of the algorithm in [3].

The formula can be easily generalized to continuous-time Markov chains. Since a Markov chain is the main model for many stochastic systems such as queueing systems, the formula developed here may have an impact in other fields, especially in the field of single sample path-based sensitivity analysis and performance optimization.

The paper is organized as follows. Section II introduces the basic concepts derived from a sample path point of view. We consider the simplest, but fundamental, case where one transition probability increases and another transition probability decreases by the same amount. We show how the sensitivity formula was derived by using intuition. Section III provides a rigorous proof for the general results where the transition matrix changes arbitrarily within the constraint of a stochastic matrix. The fundamental case discussed in Section II becomes a special case. Section IV discusses the implication of the results.

II. The Basic Concepts

In this section, by intuition we derive the performance sensitivity formula for the most fundamental case. The basic concepts are introduced. The rigorous proof of the general formula and other results will be provided in the next section.

Consider an irreducible and aperiodic Markov chain $X = \{X_n; n \geq 0\}$ on a finite state space $\mathcal{E} = \{1, 2, \cdots, M\}$ with transition probability matrix $P = [p_{ij}]_{i=1}^{M}|_{j=1}^{M}$. Let $f: \mathcal{E} \to \mathcal{R}$, where $\mathcal{R} = (-\infty, \infty)$ represents the space of real numbers, and $\pi = (\pi_1, \pi_2, \cdots, \pi_M)$ is the steady-state probability vector of X. f is called a *performance function*. The *performance measure* is defined as its expected value with respect to π

$$\eta = E(f) = \sum_{i=1}^{M} \pi_i f(i) = \pi f \tag{1}$$

where $f = (f(1), f(2), \cdots, f(M))^T$ is a column vector, and E denotes the expectation with respect to the steady-state measure π.

Assume that the transition matrix P is perturbed to $P' = [p'_{ij}]_{i=1}^{M}|_{j=1}^{M}$, where

$$p'_{ij} = \begin{cases} p_{ls} - \delta, & i = l, j = s \\ p_{lt} + \delta, & i = l, j = t \\ p_{ij}, & \text{otherwise} \end{cases} \tag{2}$$

for some arbitrarily fixed $l, s, t \in \mathcal{E}$, and $\delta > 0$ is any small real number. Let $Q = [q_{ij}]$ be a matrix with $q_{ls} = -1, q_{lt} = 1$, and $q_{ij} = 0$ for all $i \neq l$ or $j \neq s, t$. Then we have $P' = P + \delta Q$. The Markov chain with transition matrix P' is denoted as $X' = \{X'_n; n \geq$

Authorized licensed use limited to: Hong Kong University of Science and Technology. Downloaded on April 14,2021 at 06:16:50 UTC from IEEE Xplore. Restrictions apply.

$0\}$. Let $\pi' = (\pi'_1, \pi'_2, \cdots, \pi'_M)$ be the steady-state probability vector of X'. The performance measure of X' is

$$\eta' = \sum_{i=1}^{M} \pi'_i f(i) = \pi' f.$$

The derivative of η in the direction of Q is defined as

$$\frac{\partial \eta}{\partial Q} = \lim_{\delta \to 0} \frac{\eta' - \eta}{\delta}. \qquad (3)$$

Our purpose is to derive a formula for $\partial\eta/\partial Q$ based on a sample path approach.

Consider a sample path of X with a finite length $\{X_n; 0 \le n < L\}$. We define a *sample performance measure*

$$\eta_L = \frac{1}{L} \sum_{n=0}^{L-1} f(X_n). \qquad (4)$$

Since X is irreducible, aperiodic, and finite, we have [2]

$$\eta = \lim_{L \to \infty} \eta_L, \quad \text{w.p.1.}$$

That is, we can approximate η by η_L, provided L is large enough.

From (2), the state transition may change only when the Markov chain is at state l. Because $p'_{ls} = p_{ls} - \delta$ and $p'_{lt} = p_{lt} + \delta$, some transitions in X from l to s may become transitions in X' from l to t. It is obvious that when X is at state l, the probability of such transition changes is δ.

Suppose we simulate X and X' using the same sequence of random numbers. Starting from the same initial state, X and X' will be the same in an initial period; the two sample paths may differ only after the Markov chains reach state l at some time k, i.e., $X_k = l$ and $X'_k = l$. At time $k+1$, it happens that $X_{k+1} = s$ and $X'_{k+1} = t$. The effect of a state changing from s to t on a sample performance measure can be visualized as follows. For simplicity, we renumber the time indexes such that $k+1 = 0$. We run the two Markov chains X and X' with $X_0 = s$ and $X'_0 = t$. Let

$$F_L = \sum_{n=0}^{L-1} f(X_n).$$

The effect of the change from $X_0 = s$ to $X'_0 = t$ on the sample performance measure is reflected by

$$\Delta F_L = \sum_{n=0}^{L-1} [f(X'_n) - f(X_n)] \quad \text{with} \quad X_0 = s \quad \text{and} \quad X'_0 = t.$$

Its expected value is denoted as

$$d'_{st} = E\left(\sum_{n=0}^{L-1} [f(X'_n) - f(X_n)] | X_0 = s, X'_0 = t\right) \qquad (5)$$

where L is considered a very large number, X evolves according to P, and X' evolves according to P'. It is easy to see that at some point $n = L^*, X'_n$ and X_n may reach the same state, i.e., $X_{L^*} = X'_{L^*}$. After this point X'_n and X_n will behave statistically similarly. Therefore, L in (5) can be replaced by L^*, i.e.,

$$d'_{st} = E\left(\sum_{n=0}^{L^*-1} [f(X'_n) - f(X_n)] | X_0 = s, X'_0 = t\right).$$

Thus, the effect of the change from $X_0 = s$ to $X'_0 = t$ on performance terminates when the chain reaches L^*.

With the above observations, we can devise the formula for the derivative. Among the L transitions on a sample path $\{X_n; 0 \le n < L\}$, the Markov chain X visits state l, on the average, $L\pi_l$ times. Because the probability that a transition from l to s changes to a transition from l to t is δ on the sample path, there are, on the average, $L\pi_l\delta$ times when state s changes to state t. Each time the state changes from s to t, F_L changes, on the average, by the amount d'_{st}. In addition, because δ can be chosen arbitrarily small and L^* is always finite, the probability of two changes from s to t occurring within L^* state transitions is of order δ^2 and hence is negligible. This implies that the effect of each change can be treated separately. Therefore, the total change of η due to the change of P is

$$\Delta\eta = \frac{1}{L}\Delta F_L = \frac{1}{L}\{L\pi_l\delta d'_{st}\} = \pi_l \delta\, d'_{st}.$$

Finally

$$\frac{\partial \eta}{\partial Q} = \lim_{\delta \to 0} \frac{\eta' - \eta}{\delta} = \pi_l\, d_{st} \qquad (6)$$

where

$$d_{st} = E\left(\sum_{n=0}^{L^*-1} [f(X'_n) - f(X_n)] | X_0 = s, X'_0 = t\right). \qquad (7)$$

Both X and X' evolve according to P ($P' \to P$ as $\delta \to 0$).

We have obtained the new formula (6) by using a sample path argument. A mathematical proof of the formula and the equations determining $d_{ij}, i,j \in \mathcal{E}$ will be provided in the next section. The explanation in this section shows that the sample path approach may lead to new perspectives as well as new results. The theory is a counterpart of the realization theory in perturbation analysis [1], [6] in the case where there are finite changes in sample paths.

III. The Main Results

In this section, we shall rigorously establish the performance sensitivity formula for the general case where the transition matrix changes arbitrarily within the constraint of a stochastic matrix.

A. The Realization Matrix

Consider an irreducible aperiodic Markov chain X with a transition matrix P and state space $\mathcal{E} = \{1, 2, \cdots, M\}$. Let X' be another Markov chain, independent of X, with the same state space $\mathcal{E}$ and the same transition matrix P. We assume that the initial states of both chains may be different, say $X_0 = i, X'_0 = j, i, j \in \mathcal{E}$.

Define $Y_n = (X_n, X'_n), n = 0, 1, \cdots$. Then $Y = \{Y_0, Y_1, \cdots, Y_n, \cdots\}$ is a Markov chain with state space $\mathcal{E} \times \mathcal{E}$ and transition matrix $P \otimes P$, where $\otimes$ denotes the Kronecker product. Furthermore, Y is irreducible and aperiodic since X and X' are irreducible and aperiodic.

Since $\mathcal{E} \times \mathcal{E}$ is finite, it follows that all the states of Y are recurrent nonnull [2]. Thus, the first passage time from any state to any other state has a finite mean. Let $\mathcal{S} = \{(k,k): k \in \mathcal{E}\}$. Let L^* be the random variable such that at $n = L^*, Y$ reaches $\mathcal{S}$ for the first time. Then $E(L^*)$ is finite. (One reviewer pointed out that if X and X' are not independent, e.g., they are simulated by using common random numbers, then L^* may not be finite.)

Definition 1:

$$d_{ij} = E\left\{\sum_{n=0}^{L^*-1} [f(X'_n) - f(X_n)] | X_0 = i, X'_0 = j\right\}, \quad i,j = 1, 2, \cdots, M$$

is called a perturbation realization factor; the matrix $D = [d_{ij}]$ is called a realization matrix.

The meaning of the perturbation realization factor can be intuitively explained as follows: suppose that a Markov chain is perturbed

Authorized licensed use limited to: Hong Kong University of Science and Technology. Downloaded on April 14,2021 at 06:16:50 UTC from IEEE Xplore. Restrictions apply.

by some reason such that the state changes from i to j. This perturbation will affect the system performance. The average effect can be expressed as

$$E\left\{\sum_{n=0}^{\infty}[f(X_n') - f(X_n)]|X_0 = i, X_0' = j\right\}$$
$$= E\left\{\sum_{n=0}^{L^*-1}[f(X_n') - f(X_n)]|X_0 = i, X_0' = j\right\} + E\left\{\sum_{n=L^*}^{\infty}[f(X_n') - f(X_n)]|X_{L^*} = X_{L^*}'\right\}.$$

The second term is zero. Therefore, d_{ij} measures the average effect of this perturbation on the performance measure. This is in parallel to the theory for IPA [1]. We say that the effect of the perturbation from i to j has been *realized* at L^* on the sample path.

By definition, $d_{ji} = -d_{ij}$. That is

$$D^T = -D$$

where T represents the transpose of matrices or vectors.

Denote

$$F = ef^T - fe^T$$

where $e = (1, 1, \cdots, 1)^T$, and define two subspaces of $\mathcal{R}^{M\times M}$

$$\mathcal{U} = \{X \in \mathcal{R}^{M\times M}: X^T = -X\}$$
$$\mathcal{V} = \{ex^T - xe^T: x \in \mathcal{R}^M\}.$$

Clearly, $\mathcal{V} \subset \mathcal{U}, D \in \mathcal{U}$, and $F \in \mathcal{V}$.

Theorem 1: The realization matrix D is the unique solution in $\mathcal{U}$ of the Lyapunov equation

$$D - PDP^T = F. \tag{8}$$

Furthermore $D \in \mathcal{V}$.

Proof: First let us prove that D satisfies (8). For any $i, j \in \mathcal{E}$, we have

$$d_{ij} = E\left(\sum_{n=0}^{L^*}[f(X_n') - f(X_n)]|X_0 = i, X_0' = j\right)$$
$$= f(j) - f(i) + \sum_{i'=1}^{M}\sum_{j'=1}^{M} \cdot E\left(\sum_{n=1}^{L^*}[f(X_n') - f(X_n)]|X_1 = i', X_1' = j'\right) \cdot P\{X_1 = i', X_1' = j'|X_0 = i, X_0' = j\}$$
$$= f(j) - f(i) + \sum_{i'=1}^{M}\sum_{j'=1}^{M} p_{ii'}p_{jj'}\, d_{i'j'}.$$

Writing the above equations in matrix form, we obtain (8).

Consider now the Lyapunov map $\boldsymbol{L}: \mathcal{R}^{M\times M} \to \mathcal{R}^{M\times M}$ defined by

$$L(D) = D - PDP^T.$$

Simple algebra shows that $\mathcal{U}$ is an invariant subspace of $\boldsymbol{L}$. It is well known [8] that the restriction of $\boldsymbol{L}$ to $\mathcal{U}$ has eigenvalues $1 - \lambda_i(P)\lambda_j(P), 1 \le i < j \le M$, where $\lambda_i(P)$ denotes the ith eigenvalue of P. Because X is irreducible and aperiodic, P is primitive; it follows that 1 is the only eigenvalue of P with maximum modulus. Hence, this restriction is invertible, which means that for each $F \in \mathcal{U}$ there is a unique $D \in \mathcal{U}$ such that (8) is satisfied. Since $\mathcal{V}$ is also an invariant subspace of $\boldsymbol{L}$ and the restriction of $\boldsymbol{L}$ to $\mathcal{U}$ is invertible, it follows that the restriction of $\boldsymbol{L}$ to $\mathcal{V}$ is also invertible, which implies that $D \in \mathcal{V}$ for each $F \in \mathcal{V}$. □

$D \in \mathcal{V}$ implies

$$d_{ik} = d_{ij} + d_{jk}, \quad \text{for all } i, j, k \in \mathcal{E}. \tag{9}$$

Corollary 1: The unique skew-symmetric solution to (8) is given by

$$D = \sum_{n=0}^{\infty} P^n F (P^T)^n.$$

Proof: Replacing the D on the right-hand side of (8) by $F + PDP^T$ and working iteratively, we get

$$D = \sum_{n=0}^{N} P^n F (P^T)^n + P^{N+1} D (P^T)^{N+1}. \tag{10}$$

Since X is irreducible and aperiodic, we have [2]

$$\lim_{N\to\infty} P^N = e\pi.$$

Because $D^T = -D$, we have $\pi D \pi^T = 0$. Therefore

$$\lim_{N\to\infty} P^N D (P^T)^N = e\pi D \pi^T e^T = 0.$$

The corollary follows directly from this and (10). □

B. The Derivative Formula

Now we assume that P is perturbed to P' according to the following pattern:

$$P' = P + \delta Q \tag{11}$$

where $\delta > 0$ is a small real number and $Qe = 0$.

Let π and π' be the steady-state probability vectors of the Markov chain with P and P', and let $\eta = \pi f$ and $\eta' = \pi' f$ be their steady-state performance measures, respectively. The performance derivative of η in the direction of Q is defined as

$$\frac{\partial \eta}{\partial Q} = \lim_{\delta\to 0} \frac{\eta' - \eta}{\delta}.$$

The derivatives of other quantities are defined in a similar way.

Taking derivatives of the both sides of $\pi P = \pi$ in the direction of Q, we have

$$\frac{\partial \pi}{\partial Q}(I - P) = \pi \frac{\partial P}{\partial Q} = \pi Q.$$

It follows from $(\partial\pi/\partial Q)e = 0$ that

$$\frac{\partial \pi}{\partial Q}(I - P + e\pi) = \pi Q.$$

Since the *fundamental matrix* [7] $I - P + e\pi$ is invertible, we have

$$\frac{\partial \pi}{\partial Q} = \pi Q (I - P + e\pi)^{-1}. \tag{12}$$

Theorem 2:

$$\frac{\partial \eta}{\partial Q} = \pi Q D^T \pi^T.$$

Authorized licensed use limited to: Hong Kong University of Science and Technology. Downloaded on April 14,2021 at 06:16:50 UTC from IEEE Xplore. Restrictions apply.

Proof: From (12)

$$\frac{\partial \eta}{\partial Q} = \frac{\partial \pi}{\partial Q} f = \pi Q (I - P + e\pi)^{-1} f. \quad (13)$$

Since $D = F + PDP^T, \pi e = 1$, and $\pi P = \pi$, we have

$$D^T \pi^T = F^T \pi^T + PD^T P^T \pi^T = f - (\pi f)e + PD^T \pi^T.$$

Furthermore

$$(I - P + e\pi) D^T \pi^T = (\pi D^T \pi^T)e + f - (\pi f)e.$$

Now we have

$$(I - P + e\pi)^{-1} f = D^T \pi^T + (\pi f - \pi D^T \pi^T)(I - P + e\pi)^{-1} e = D^T \pi^T + (\pi f - \pi D^T \pi^T)e.$$

The last equation is due to $(I - P + e\pi)e = e$. Therefore, from (13) and $Qe = 0$

$$\frac{\partial \eta}{\partial Q} = \pi Q [D^T \pi^T + (\pi f - \pi D^T \pi^T)e] = \pi Q D^T \pi^T.$$

This concludes the proof. □

For the case discussed in Section II, we have $q_{ls} = -1, q_{lt} = 1$, and $q_{ij} = 0$ for $i \neq l$ or $j \neq l, s$. We have

$$\frac{\partial \eta}{\partial Q} = \pi_l \sum_{j=1}^{M} \sum_{k=1}^{M} \pi_j q_{lk} \, d_{jk} = \pi_l \sum_{j=1}^{M} \pi_j (q_{ls} \, d_{js} + q_{lt} \, d_{jt}) = \pi_l \sum_{j=1}^{M} \pi_j (d_{jt} - d_{js}) = \pi_l \, d_{st}. \quad (14)$$

The last equation is due to (9). Thus, $\partial \eta / \partial Q$ equals the expected value of d_{st}.

Finally, the derivative $\partial \eta / \partial Q$ is similar to the directional derivative in calculus. Therefore, if Q is multiplied by a constant, so is $\partial \eta / \partial Q$. Of course, we can normalize Q so that the derivative depends on only the direction of the changes.

IV. Discussions

Using a sample path approach, we have derived a new formula for the performance sensitivity of Markov chains. A distinguished feature of this formula is that the quantities involved can be estimated by analyzing a single sample path of a Markov chain.

To estimate d_{st} defined in (7) on a sample path of a Markov chain X, we may proceed as follows. First, find an $X_i = s$ and an $X_j = t$ on the sample path. Then record $\Sigma_{n=i}^{i+L} f(X_n)$ and $\Sigma_{n=j}^{j+L} f(X_n)$ until $X_{i+L} = X_{j+L}$. The average of $\Sigma_{n=i}^{i+L} f(X_n) - \Sigma_{n=j}^{j+L} f(X_n)$ is an estimate of d_{st}. This demonstrates the important principle that a single sample path contains all information of performance sensitivity.

Reference [3] proposed a simulation algorithm which in fact estimates the realization factors. In Dai's method, an additional simulation for a Markov chain X' with the initial state t is carried out when X reaches s. The additional simulation stops when X' and X reach the same state. The results obtained in this paper thus provide a clear interpretation of Dai's method as well as a concise proof of the algorithm in [3].

To develop single sample path-based algorithms using the formula for estimating performance sensitivities of practical systems is an ongoing research topic.

References

[1] X. R. Cao, *Realization Probabilities: The Dynamics of Queueing Systems.* New York: Springer-Verlag, 1994.

[2] E. Çinlar, *Introduction to Stochastic Processes.* Englewood Cliffs, NJ: Prentice-Hall, 1975.

[3] L. Y. Dai, "A consistent algorithm for derivative estimation of Markov chains," in *Proc. 33rd IEEE Conf. Decision Contr.*, 1994.

[4] L. Y. Dai and Y. C. Ho, "Structural infinitesimal perturbation analysis (SIPA) for derivative estimation of discrete event dynamic systems," *IEEE Trans. Automat. Contr.*, to appear.

[5] P. Glasserman, *Gradient Estimation Via Perturbation Analysis.* Boston, MA: Kluwer, 1991.

[6] Y. C. Ho and X. R. Cao, *Perturbation Analysis of Discrete-Event Dynamic Systems.* Boston, MA: Kluwer, 1991.

[7] J. G. Kemeny and J. L. Snell, *Finite Markov Chains.* New York: Van Nostrand-Reinhold, 1960.

[8] M. Marcus, *Finite Dimensional Multilinear Algebra—Part I.* New York: Marcel Dekker, 1973.

An Alternate Calculation of the Discrete-Time Kalman Filter Gain and Riccati Equation Solution

Robert Leland

***Abstract*— We describe an algorithm to calculate the steady-state Kalman filter gain and Riccati equation solution for a discrete-time Kalman filter. Our algorithm makes use of an approximate autoregressive model for the one-step predictor and only requires the solutions to linear equations. All of the nonlinear calculations can be made explicitly.**

I. Introduction

We consider a new algorithm for calculating the steady-state Kalman filter gain and error covariance for the discrete-time filter. Our algorithm only requires (in principle) the solution of linear equations, and all of the nonlinear calculations are explicit.

Suppose we have the discrete-time stochastic system

$$x_{k+1} = \Phi x_k + w_k$$
$$z_k = C x_k + v_k$$

where Φ is a discrete-time stable $n \times n$ matrix, w_k and v_k are independent zero mean Gaussian white noise processes with $E[w_k w_k^T] = Q, E[v_k v_k^T] = R$, where $Q \geq 0$, and $R > 0$. We also assume that (C, Φ) is observable and Φ is nonsingular. This last assumption is

Manuscript received August 29, 1996. This work was supported by the NSF under Grant ECS-9211596.

The author is with the Department of Electrical Engineering, University of Alabama, Tuscaloosa, AL 35487 USA (e-mail: rleland@smisl.carl.ua.edu).

Publisher Item Identifier S 0018-9286(96)08447-4.

0018–9286/96$05.00 © 1996 IEEE

Authorized licensed use limited to: Hong Kong University of Science and Technology. Downloaded on April 14,2021 at 06:16:50 UTC from IEEE Xplore. Restrictions apply.