These new stability conditions are related to the use of piecewise quadratic Lyapunov functions, but do not require an explicit partitioning of the state space. Instead, the appropriate partition falls out of the necessary conditions from the saturation operator formulated as an optimization. Furthermore, it should be clear that this approach can easily be extended to the analysis of general piecewise linear systems. When tested versus previous results, the conditions in this paper were found to match even the Zames–Falb stability conditions for an example where both the circle and Popov criteria fail to produce strong results. An important feature of the proposed method is its ability to capture the distinction between a deadzone and a saturation. This is achieved because a piecewise quadratic Lyapunov function is used in the stability analysis. Such a difference cannot be represented by the Zames–Falb criterion within the multiplier analysis context.

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Optimal Tracking Performance: Preview Control and Exponential Signals

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***Abstract*—In this note, we study tracking performance limitation problems. Two issues are addressed, concerning how earlier results developed elsewhere may be extended to more general classes of reference signals, and how tracking performance may be further improved beyond that offered by feedback control. Toward these issues we consider exponentially increasing reference inputs and examine the use of preview control for tracking. We take an optimal interpolation approach, and our purpose is to develop analytical expressions and conceptual insight which will aid in the understanding of these issues. To this effect, we derive explicit expressions for the optimal tracking error, either as exact solutions or bounds. It is found that for the exponential signals the earlier results can be directly extended, and similar conclusive statements can be drawn. It is also shown that in general preview can be used to advantage for improving tracking performance, especially in countering the effect resulted from plant nonminimum phase zeros.**

***Index Terms*—Exponential signals, nonminimum phase zeros, preview control, tracking performance, unstable poles.**

I. Introduction

The ability of tracking command input signals is a primary criterion for assessing the performance of feedback control systems and indeed it constitutes a primary objective in control system design. As such, optimal tracking problems have over the years received a considerable amount of research interest. While in many such problems a main objective is to design an optimal compensator to minimize tracking error, which from a numerical computation viewpoint can be tackled using standard techniques and routines, and thus is considered a resolved issue, more recent attention has been focused on the understanding of the inherent limitation on the best tracking performance achievable via feedback. This has led to several important discoveries. Among the notable issues and results are cheap LQR control [11], servomechanism problems [17], and optimal tracking control [5], [14], [16], [18]. By now it is generally known that in the full generality of *causal* feedback compensation, i.e., when a two-parameter causal feedback control scheme is employed, the best achievable tracking performance is limited, and in fact is only limited, by the nonminimum phase characteristics of plant [5]; here by the latter we mean both the nonminimum phase zeros as well as time delays in the plant. Consequently, such characteristics impose an intrinsic barrier which in no way may be surpassed by causal feedback alone, in that the tracking accuracy cannot be further improved by use of any causal feedback controller.

One of the main issues to be investigated in this note dwells on the use of *noncausal* actions for tracking. More specifically, can

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noncausal action, where it is allowed and implementable, aid in improving tracking performance? In light of the aforementioned limitation of causal feedback, this contemplation is warranted, and it leads us to the use of preview control. Preview control is a means of using the future information of the reference input for control, and in the context of tracking, it amounts to tracking a delayed reference. In essence, the compensator, which itself is causal, must then act on a time-advanced signal, and hence involves a noncausal operation on the reference signal. This control strategy has most notably found its utility in various tracking problems (see, e.g., [8], [10], [19], [20]), which is known to be useful in improving tracking performance; this has been shown explicitly in [7] for single-input–single-output (SISO) discrete-time systems. Indeed, since in a tracking problem the reference signal is typically specified *a priori*, a pure time advance would introduce no error nor distortion. Thus, in a preview control scheme, while tracking the true (albeit delayed) reference signal, the system can exploit fully to advantage the known future information of the reference signal. It will be seen that this does help reduce the tracking error. Of course, the preview tracking scheme is possible only when the future information on the reference is made available, so that a noncausal operation may be peformed. This clearly is the case when tracking a pre-specified signal.

Another purpose of this note lies in our attempt to extend the current work to more general and perhaps more problematic signals. In the study of tracking performance, it has been customary to consider step references. This simplicity enables the derivation of explicit expressions relating tracking error to plant nonminimum phase zeros [2], [5], [14], [17], [18], [22], thus displaying in a clear manner how the error may be affected by such zeros. Accordingly, much of the understanding on tracking performance limitation draws upon analysis of these expressions, though similar results have also been obtained for ramp and sinusoidal signals [6], [17]. In the present paper, we consider exponentially increasing reference signals. This consequently enables us to gain additional insight into the problem and extend the existing knowledge further beyond.

Our development also offers an alternative, interpolation-based perspective to optimal tracking problems. Unlike in the previous work, we formulate and solve the problem directly as an optimal $\mathcal{H}_2$ interpolation problem. In other words, the optimal tracking performance is obtained by computing the minimal $\mathcal{H}_2$ norm of a certain function analytic in the right half of the complex plane, subject to constraints imposed by the plant nonminimum phase zeros and unstable poles. This approach bypasses the usual controller parameterization and model matching problem, and appears to be conceptually simpler. Clearly, it also has the flavor of similar work on performance limits quantified under an $\mathcal{H}_\infty$ criterion [4], [9], [21].

Finally, we should point out that the optimal tracking problems under consideration herein can all be solved numerically as an $\mathcal{H}_2$ optimal control problem. In particular, it can be cast as a singular $\mathcal{H}_2$ control problem with unstable weighting functions; these problems have been studied in, e.g., [12], [13]. We emphasize, however, that our intention is not to seek numerical solutions. Instead, we are interested in explicit, analytical expressions of the minimal tracking error, and further, in how intrinsic system properties such as nonminimum phase zeros may constrain the best achievable tracking performance. For this purpose, we shall sometimes trade the exactness of the results for their conceptual appeal, by deriving bounds or examining limiting cases.

II. Preliminaries

We begin with the notation used throughout this paper. For any complex number z, we denote its complex conjugate by $\overline{z}$. For any vector u, we denote its conjugate transpose by u^{H}. For any signal $u(t)$, we denote its Laplace transform by $\widehat{u}(s)$. The conjugate transpose of a matrix A is denoted by A^{H}. The trace of a matrix A is denoted by $\mathrm{Tr}(A)$, and $\overline{\sigma}(A)$ denotes the largest singular value of A. For a Hermitian matrix A, we write $A \geq 0$ if A is nonnegative definite, and $A > 0$ if it is positive definite. All the vectors and matrices involved in the sequel are assumed to have compatible dimensions, and for simplicity, their dimensions are omitted. Let the open right half plane be denoted by $\mathbb{C}_+ := \{s : \mathrm{Re}(s) > 0\}$, and the imaginary axis by $\mathbb{C}_0$. Moreover, let $\|\cdot\|$ denote the Euclidean vector norm. Define

$$\mathcal{L}_2 := \left\{ f\colon f(s) \text{ measurable in } \mathbb{C}_0, \; \|f\|_2 := \left(\frac{1}{2\pi}\int_{-\infty}^{\infty} \|f(j\omega)\|^2 d\omega\right)^{1/2} < \infty \right\}$$

and

$$\mathcal{H}_2 := \left\{ f\colon f(s) \text{ analytic in } \mathbb{C}_+, \; \|f\|_2 := \left(\sup_{\sigma>0} \frac{1}{2\pi}\int_{-\infty}^{\infty} \|f(\sigma + j\omega)\|^2 d\omega\right)^{1/2} < \infty \right\}.$$

Note that for each of the normed spaces $\mathcal{L}_2$ and $\mathcal{H}_2$, we have used the same notation $\|\cdot\|_2$ to denote the corresponding norm. However, use of each of these norms will be clear from the context. Finally, we define the angle between the directions spanned by two vectors w and v by

$$\cos\angle(w, v) := \frac{|w^{\mathrm{H}} v|}{\|w\|\|v\|}.$$

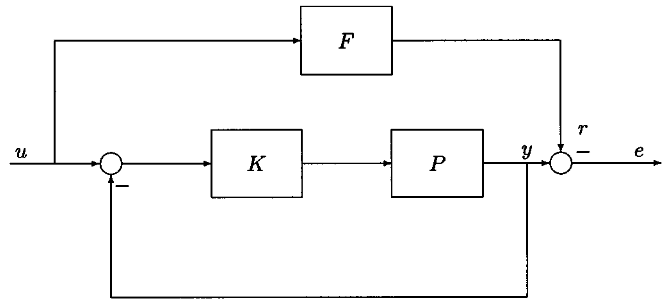

Fig. 1. The tracking structure.

Our tracking problem is schematically represented by the linear time-invariant system depicted in Fig. 1. In this setup, P denotes the plant model and K the compensator, whose transfer function matrices are $P(s)$ and $K(s)$, respectively. We assume that $P(s)$ and $K(s)$ are both rational transfer function matrices. The signals u and y represent respectively the reference input and the system output. The feedthrough transfer function matrix $F(s)$ implements the preview strategy. More generally, it may also be viewed as a prefilter. The output signal y is to track a *filtered* signal r, which is generated through the filter F. The tracking quality is measured by the error signal e. We shall assume throughout that $F(s)$ is stable. Let the system's complementary sensitivity function be defined as

$$T(s) = P(s)K(s)[I + P(s)K(s)]^{-1}.$$

Then the Laplace transform of the error signal $e(t)$ can be expressed

$$\widehat{e}(s) = [T(s) - F(s)]\widehat{u}(s).$$

We use the $\mathcal{L}_2$ norm of $\widehat{e}(s)$ to measure the tracking performance, and we are interested in the best possible tracking error achievable by all feedback compensators that stabilize the closed-loop system. Specifically, we want to determine

$$J_2(F) := \inf \{\|[T(s) - F(s)]\widehat{u}(s)\|_2 : K \text{ stabilizes } P\}.$$

Thus, for any given F, $J_2(F)$ provides the intrinsic limit to the tracking performance which cannot be further reduced by feedback design.

Intuitively, if one chooses to introduce an attenuating filter in the feedforward path and track the filtered signal, the tracking error may be reduced, as the system attempts to track a signal of an attenuated amplitude. Such a filtered signal, however, will be distorted in general. Of particular interest in this note are the following two cases for $F(s)$.

i) $F(s) = I$. This corresponds to the standard tracking scheme. The output y is to track the reference signal u directly.
ii) $F(s) = \operatorname{diag}\left(e^{-T_1 s}, \ldots, e^{-T_m s}\right)$. This corresponds to preview tracking scheme. The output y is to track a delayed but otherwise distortionless reference signal, each of whose components may be delayed by a different amount of time T_i. In effect, it amounts to advancing, or "previewing" the input to K relatively to the reference u, and, hence, advancing the output y, so that the advanced output may better track the original reference input u.

It is worth noting that in both cases $F(s)$ are allpass, and hence neither attenuation nor distortion will be incurred on the reference signal. This insures that the very original goal of tracking be met: the output y tracks asymptotically the reference u. We note that under the $\mathcal{L}_2$ error criterion Case i) has been well studied [5], [14], but Case ii) is new. Accordingly, the latter will receive primary attention in the sequel. We also point out that the tracking scheme represented by Fig. 1 utilizes a one-parameter feedback structure. In this case, the tracking performance depends on both the nonminimum phase zeros and the unstable poles of $P(s)$. More generally, a two-parameter feedback controller may be employed, with which the tracking performance will only be affected by the plant nonminimum phase zeros [5].

The main technical tool to be used in our development is the theory of analytic function interpolation. In particular, the following necessary and sufficient condition concerning the $\mathcal{H}_2$ optimal interpolation problem will play a pivotal role. The result can be found in [15].

Lemma 1: Consider two sets of distinct points $z_i \in \mathbb{C}_+$, $i = 1, \ldots, m$ and $p_i \in \mathbb{C}_+$, $i = 1, \ldots, n$. Assume that $z_i \neq p_j$ for any i and j. Then, there exists a rational matrix function $H(s)$ such that i) $H(s)$ is analytic in $\mathbb{C}_+$, ii) $\|H(s)\|_2 \leq \gamma$, and iii) $H(s)$ satisfies the conditions

$$x_i^{\mathrm{H}} H(z_i) = y_i^{\mathrm{H}}, \quad i = 1, \ldots, m \tag{2.1}$$
$$H(p_i) u_i = v_i, \quad i = 1, \ldots, n \tag{2.2}$$

for some vector sequences x_i, y_i, $i = 1, \ldots, m$ and u_i, v_i, $i = 1, \ldots, n$, of compatible dimensions, if and only if

$$\operatorname{Tr}\left(Y Q_x^{-1} Y^{\mathrm{H}}\right) + \operatorname{Tr}\left([V + X Q_x^{-1}(Q_{yu} - Q_{xv})] Q_u^{-1} \left[V + X Q_x^{-1}(Q_{yu} - Q_{xv})\right]^{\mathrm{H}}\right) \leq \gamma^2$$

where

$$Q_x := \left[\frac{x_i^{\mathrm{H}} x_j}{z_i + \overline{z}_j}\right] \quad Q_u := \left[\frac{u_i^{\mathrm{H}} u_j}{\overline{p}_i + p_j}\right]$$
$$Q_{yu} := \left[\frac{y_i^{\mathrm{H}} u_j}{z_i - p_j}\right] \quad Q_{xv} := \left[\frac{x_i^{\mathrm{H}} v_j}{z_i - p_j}\right]$$
$$V := [v_1 \quad v_2 \quad \ldots \quad v_n]$$
$$X := [x_1 \quad x_2 \quad \ldots \quad x_m]$$
$$Y := [y_1 \quad y_2 \quad \ldots \quad y_m].$$

This problem, while similar to the well-known Nevanlinna–Pick interpolation problem (see, e.g., [1]), amounts to determining an analytic function which satisfies a set of prescribed interpolation constraints and whose $\mathcal{H}_2$ norm is bounded.

III. Main Results

Throughout this note, we consider the exponentially increasing reference signal

$$\widehat{u}(s) = \frac{v}{s - \sigma} \tag{3.1}$$

where $\sigma \geq 0$, and v is a unitary constant vector. We shall make the following assumption.

Assumption 1: $P(s)$ is right-invertible and has no zero at $s = \sigma$. Here for a right-invertible $P(s)$, a point z is said to be a zero of $P(s)$ if $w^{\mathrm{H}} P(z) = 0$ for some unitary vector w, and w is referred to as the output zero direction vector associated with z. This assumption is necessary, for otherwise the tracking error cannot be finite, and hence the output will be unable to track the reference input. It is also sufficient, since $P(\sigma)$ is of full-row rank and $F(\sigma)v$ lies necessarily in the column space of $P(\sigma)$. By a proper design of $K(s)$, it is then possible to insure that

$$[F(s) - T(s)]\widehat{u}(s) = \frac{[F(s) - T(s)]v}{s - \sigma} \in \mathcal{H}_2.$$

We note that the assumption is reminiscent of the well-known internal model principle [14].

We now cast the optimal tracking problem as one of $\mathcal{H}_2$ optimal interpolation. Consider the feedback system in Fig. 1. We begin with the following well-known interpolation constraints on the sensitivity and complementary sensitivity functions, imposed by the plant nonminimum phase zeros and unstable poles (see, e.g., [5]).

Lemma 2: Suppose that $p \in \mathbb{C}_+$ is a pole of $P(s)$ with input pole direction vector η, and $z \in \mathbb{C}_+$ a zero of $P(s)$ with output zero direction vector w. Then in order for the closed-loop system to be stable, the following conditions must hold:

$$S(p)\eta = 0, \qquad T(p)\eta = \eta$$
$$w^{\mathrm{H}} S(z) = w^{\mathrm{H}}, \qquad w^{\mathrm{H}} T(z) = 0.$$

Here the zero and pole direction vectors w and η are unitary vectors, $\|w\| = \|\eta\| = 1$. Thus, to compute the minimal tracking error $J_2(F)$, it suffices to find the minimal norm of $[F(s) - T(s)]\widehat{u}(s)$ so that it is in $\mathcal{H}_2$ and meets the above interpolation requirements.

A. Stable Plants

We shall mainly focus on stable plants. Unstable plants will remain to be of interest but will be deferred to the next subsection. Under this premise, the minimal tracking error $J_2(F)$ has a simpler expression. Suppose first that $F(s)$ is a rational transfer function.

Theorem 1: Let $\widehat{u}(s)$ be given by (3.1). Suppose that $P(s)$ is stable and has distinct zeros $z_i \in \mathbb{C}_+$, $i = 1, \ldots, m$, with output zero direction vectors w_i. Then, under Assumption 1 and for any stable proper rational $F(s)$

$$J_2^2(F) = Y Q_w^{-1} Y^{\mathrm{H}} \tag{3.2}$$

where

$$Y^{\mathrm{H}} := \begin{bmatrix} \frac{w_1^{\mathrm{H}} F(z_1) v}{z_1 - \sigma} \\ \vdots \\ \frac{w_m^{\mathrm{H}} F(z_m) v}{z_m - \sigma} \end{bmatrix}, \quad Q_w := \begin{bmatrix} \frac{w_1^{\mathrm{H}} w_1}{z_1 + \overline{z}_1} & \cdots & \frac{w_1^{\mathrm{H}} w_m}{z_1 + \overline{z}_m} \\ \vdots & \cdots & \vdots \\ \frac{w_m^{\mathrm{H}} w_1}{z_m + \overline{z}_1} & \cdots & \frac{w_m^{\mathrm{H}} w_m}{z_m + \overline{z}_m} \end{bmatrix}.$$

Proof: Let $H(s) = [F(s) - T(s)]\widehat{u}(s)$, with $\hat{u}(s)$ given by (3.1). Then, for the closed-loop system to be stable, it is necessary that

$$w_i^{\mathrm{H}} H(z_i) = \frac{w_i^{\mathrm{H}} F(z_i) v}{z_i - \sigma}, \quad i = 1, \ldots, m. \tag{3.3}$$

In light of Lemma 1, it follows that:

$$\begin{aligned} J_2^2(F) = &\inf \Bigg\{ \gamma^2 : H(s) \text{ analytic in } \mathbb{C}_+, \|H(s)\|_2 \le \gamma, \\ &\qquad w_i^{\mathrm{H}} H(z_i) = \frac{w_i^{\mathrm{H}} F(z_i) v}{z_i - \sigma}, \quad i = 1, \ldots, m \Bigg\} \\ = &\inf \left\{ \gamma^2 : \mathrm{Tr}\left(Y Q_w^{-1} Y^{\mathrm{H}} \right) \le \gamma^2 \right\} \\ = & Y Q_w^{-1} Y^{\mathrm{H}}. \end{aligned}$$

Thus, the proof is completed. ■

The following lower bound on $J_2(F)$ is immediate from Theorem 1.

Corollary 1: Under the conditions in Theorem 1, for any $i = 1, \ldots, m$

$$J_2^2(F) \ge \frac{2\mathrm{Re}(z_i)}{|z_i - \sigma|^2} \left| w_i^{\mathrm{H}} F(z_i) v \right|^2. \tag{3.4}$$

In particular

$$J_2^2(I) \ge \frac{2\mathrm{Re}(z_i)}{|z_i - \sigma|^2} \cos^2 \angle (w_i, v). \tag{3.5}$$

Proof: This follows readily by manipulating the condition (3.2). Indeed, we may rewrite

$$J_2^2(F) = \inf \left\{ \gamma^2 : Q_w - \frac{1}{\gamma^2} Y^{\mathrm{H}} Y \ge 0 \right\}.$$

By examining the diagonal elements of $Y^{\mathrm{H}} Y$, it follows that in order to meet the condition

$$Q_w - \frac{1}{\gamma^2} Y^{\mathrm{H}} Y \ge 0$$

it is necessary that

$$\frac{1}{2\mathrm{Re}(z_i)} - \frac{1}{\gamma^2} \frac{\left| w_i^{\mathrm{H}} F(z_i) v \right|^2}{|z_i - \sigma|^2} \ge 0, \quad i = 1, \ldots, m.$$

Thus

$$J_2^2(F) \ge \inf \left\{ \gamma^2 : \frac{1}{2\mathrm{Re}(z_i)} - \frac{1}{\gamma^2} \frac{\left| w_i^{\mathrm{H}} F(z_i) v \right|^2}{|z_i - \sigma|^2} \ge 0 \right\}.$$

This gives the inequality (3.4). ■

Corollary 1 shows that when there exists a zero z_i of $P(s)$ near $s = \sigma$, a large tracking error will result, whenever $|w_i F(z_i) v|$ is not small. In what follows we show that the tracking performance may be improved with the help of preview, for which $F(s)$ is selected specifically as

$$\Lambda(s) := \mathrm{diag}\left(e^{-T_1 s}, \ldots, e^{-T_l s} \right), \quad T_i \ge 0.$$

It will be instructive to consider first the special case where $P(s)$ has a single right-half plane zero.

Theorem 2: Let $\widehat{u}(s)$ be given by (3.1). Suppose that $P(s)$ is stable and has only one zero $z \in \mathbb{C}_+$, with output zero direction vector w. Then under Assumption 1

$$\begin{aligned} J_2^2(\Lambda) = &\frac{2\mathrm{Re}(z)}{|z - \sigma|^2} \left(\sum_{i=1}^{l} |v_i|^2 e^{-2\mathrm{Re}(z) T_i} \right) \\ &\times \cos^2 \angle \left(w, \frac{\Lambda(z) v}{\|\Lambda(z) v\|} \right). \end{aligned} \tag{3.6}$$

In particular, if $T_1 = \cdots = T_l = T$, then

$$J_2^2(\Lambda) = e^{-2\mathrm{Re}(z) T} \frac{2\mathrm{Re}(z)}{|z - \sigma|^2} \cos^2 \angle (w, v). \tag{3.7}$$

It is clear from Theorem 2 that the tracking error depends on three factors. The negative effect of the plant nonminimum phase zero z is present for all $F(s)$, which requires no further elaboration. The effect of time delays, which arises due to the preview action, however, is encouraging. Theorem 2 shows that the tracking error can be reduced in exponential proportion to the values of T_i. In light of (3.6), it becomes clear that preview control does help improve tracking performance, even for a moderately short preview time; this is especially the case for zeros far away from the imaginary axis. Furthermore, the effect can be especially visible when T_i are selected in accordance with the values of v_i. Clearly, for a higher amplitude $|v_i|$, a longer preview time is advised. This, of course, is consistent with our intuition. Moreover, yet one more additional term in (3.6) points to the subtlety of the preview effect: it also depends on how $\Lambda(z)$ may reshape the relative orientation of the input and zero directions. When in the limiting case a uniform preview time is adopted, the mutual orientation of the two directions is unchanged, and the expression (3.7) shows that the error then depends on the principal angle between the two directions. More generally, however, different preview times in different channels will alter this orientation, thus affecting the tracking performance directly. Finally, we should also point out that preview tracking is generally performed on a prefiltered signal. This can be readily accommodated in the present formulation by passing the reference input through a lowpass filter prior to preview action; in the expressions of the tracking error (3.6) and (3.7), it amounts to including an additional weighting factor related to the transfer function magnitude evaluated at z.

We need a number of preliminary lemmas in order to prove Theorem 2, the first of which can be found in [3, p. 67].

Lemma 3: Let

$$f_n(s) := \left(1 + \frac{s}{n}\right)^n.$$

Then $f_n(s) \to e^s$ uniformly on any compact set as $n \to \infty$.

This fact leads instantly to the following lemma.

Lemma 4: Define

$$F_n(s) := \begin{bmatrix} \left(\frac{1 - \frac{sT_1}{2n}}{1 + \frac{sT_1}{2n}} \right)^n & & \\ & \ddots & \\ & & \left(\frac{1 - \frac{sT_l}{2n}}{1 + \frac{sT_l}{2n}} \right)^n \end{bmatrix}.$$

Then $F_n(s) \to \Lambda(s)$ uniformly on any compact set as $n \to \infty$.

Since $F_n(s)$ is a stable proper rational function, the measure $J_2(F_n)$ can be computed using Theorem 1.

Lemma 5:

$$\lim_{n \to \infty} J_2(F_n) = J_2(\Lambda).$$

Proof: Since $[F_n(s) - T(s)]\widehat{u}(s) \in \mathcal{L}_2$, for any $\epsilon_1 > 0$, there exists an $R > 0$ such that

$$\begin{aligned} &\int_{-R}^{R} \|[F_n(j\omega) - T(j\omega)]\widehat{u}(j\omega)\| d\omega - \epsilon_1 \\ &\le \|[F_n(s) - T(s)]\widehat{u}(s)\|_2 \\ &\le \int_{-R}^{R} \|[F_n(j\omega) - T(j\omega)]\widehat{u}(j\omega)\| d\omega + \epsilon_1. \end{aligned}$$

Since $F_n(j\omega) \to \Lambda(s)$ uniformly for all $\omega \in [-R, R]$, it follows that for any $R > 0$,

$$\lim_{n\to\infty} \int_{-R}^{R} \|[F_n(j\omega) - T(j\omega)]\hat{u}(j\omega)\| d\omega = \int_{-R}^{R} \|[\Lambda(j\omega) - T(j\omega)]\hat{u}(j\omega)\| d\omega.$$

That is, for any $\epsilon_2 > 0$, there exists an integer $N > 0$ such that for $n \geq N$

$$\begin{aligned} \int_{-R}^{R} \|[\Lambda(j\omega) - T(j\omega)]\hat{u}(j\omega)\| d\omega - \epsilon_2 &\leq \int_{-R}^{R} \|[F_n(j\omega) - T(j\omega)]\hat{u}(j\omega)\| d\omega \\ &\leq \int_{-R}^{R} \|[\Lambda(j\omega) - T(j\omega)]\hat{u}(j\omega)\| d\omega + \epsilon_2. \end{aligned}$$

These together suggest that for any $\delta > 0$, there exists an $R > 0$, and an $N > 0$ such that for $n \geq N$,

$$\begin{aligned} \int_{-R}^{R} \|[\Lambda(j\omega) - T(j\omega)]\hat{u}(j\omega)\| d\omega - \delta &\leq \|[F_n(s) - T(s)]\hat{u}(s)\|_2 \\ &\leq \int_{-R}^{R} \|[\Lambda(j\omega) - T(j\omega)]\hat{u}(j\omega)\| d\omega + \delta \end{aligned}$$

or equivalently

$$\lim_{n\to\infty} \|[F_n(s) - T(s)]\hat{u}(s)\|_2 = \|[\Lambda(s) - T(s)]\hat{u}(s)\|_2.$$

Since this holds for any controller such that $[F_n(s) - T(s)]\hat{u}(s) \in \mathcal{L}_2$ and $[\Lambda(s) - T(s)]\hat{u}(s) \in \mathcal{L}_2$, the result follows. ■

The proof for Theorem 2 can then be completed by invoking Theorem 1 to obtain $J_2(F_n)$, and subsequently taking the limit of $J_2(F_n)$ with $n \to \infty$, using Lemma 4.

More generally, it remains possible to draw the same conceptual statement when $P(s)$ has more than one right half plane zero. We provide a number of bounds to this effect. The proof of the following corollary is rather straightforward and is thus omitted.

Corollary 2: Under the conditions in Theorem 1, for any $i = 1, \ldots, m$

$$J_2^2(\Lambda) \geq \frac{2\mathrm{Re}(z_i)}{|z_i - \sigma|^2} \left(\sum_{k=1}^{l} |v_k|^2 e^{-2\mathrm{Re}(z_i)T_k} \right) \times \cos^2 \angle \left(w_i, \frac{\Lambda(z_i)v}{\|\Lambda(z_i)v\|} \right) \tag{3.8}$$

and

$$J_2^2(\Lambda) \leq \overline{\sigma}\left(Q_w^{-1}\right) \sum_{i=1}^{m} \frac{1}{|z_i - \sigma|^2} \left(\sum_{k=1}^{l} |v_k|^2 e^{-2\mathrm{Re}(z_i)T_k} \right) \times \cos^2 \angle \left(w_i, \frac{\Lambda(z_i)v}{\|\Lambda(z_i)v\|} \right). \tag{3.9}$$

The upper bound (3.9) confirms that in general preview can be used to improve tracking performance. The lower bound (3.8), on the other hand, is useful for estimating the required preview time *a priori* to keep the tracking error under a prescribed threshold.

We conclude this subsection by presenting below an explicit expression of the tracking error for the standard tracking problem ($F(s) = I$), for single-input single-output plants. The result extends the previous work on tracking step signals [5], [14], [17], demonstrating explicitly the difficulty in tracking an exponentially increasing signal.

Theorem 3: Let $P(s)$ be a scalar transfer function, and let $\hat{u}(s)$ be given by (3.1) with $v = 1$. Let also $\sigma > 0$. Suppose that $P(s)$ is stable and has distinct zeros $z_i \in \mathbb{C}_+$, $i = 1, \ldots, m$. Then under Assumption 1

$$J_2^2(I) = \frac{1}{2\sigma} \left(\prod_{i=1}^{m} \left| \frac{\sigma + \overline{z}_i}{\sigma - z_i} \right|^2 - 1 \right). \tag{3.10}$$

Proof: Note first that for a SISO system

$$Q_w = Q := \begin{bmatrix} \frac{1}{z_1+\overline{z}_1} & \cdots & \frac{1}{z_1+\overline{z}_m} \\ \vdots & \cdots & \vdots \\ \frac{1}{z_m+\overline{z}_1} & \cdots & \frac{1}{z_m+\overline{z}_m} \end{bmatrix}$$

and

$$J_2^2(I) = \left[\frac{1}{\overline{z}_1-\sigma} \quad \cdots \quad \frac{1}{\overline{z}_m-\sigma} \right] Q^{-1} \begin{bmatrix} \frac{1}{z_1-\sigma} \\ \cdots \\ \frac{1}{z_m-\sigma} \end{bmatrix}. \tag{3.11}$$

Next, consider the functions

$$f(s) = \prod_{i=1}^{m} \frac{s - z_i}{s + \overline{z}_i} \quad f_j(s) = \prod_{\substack{i=1 \\ i\neq j}}^{m} \frac{s - z_i}{s + \overline{z}_i} \quad h(s) = \frac{f(s)}{s - \sigma}.$$

The function $h(s)$ can be expanded via partial fraction as

$$h(s) = \sum_{i=1}^{m} \frac{2\mathrm{Re}(z_i) f_i(-\overline{z}_i)}{\overline{z}_i + \sigma} \frac{1}{s + \overline{z}_i} + f(\sigma)\frac{1}{s - \sigma}.$$

Since $h(z_i) = 0$, we obtain

$$Q \begin{bmatrix} \frac{2\mathrm{Re}(z_1)}{\overline{z}_1+\sigma} f_1(-\overline{z}_1) \\ \vdots \\ \frac{2\mathrm{Re}(z_m)}{\overline{z}_m+\sigma} f_m(-\overline{z}_m) \end{bmatrix} + f(\sigma) \begin{bmatrix} \frac{1}{z_1-\sigma} \\ \vdots \\ \frac{1}{z_m-\sigma} \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}.$$

In light of (3.11), this gives rise to

$$\begin{aligned} J_2^2(I) &= -\frac{1}{f(\sigma)} \left[\frac{1}{\overline{z}_1-\sigma} \quad \cdots \quad \frac{1}{\overline{z}_m-\sigma} \right] \begin{bmatrix} \frac{2\mathrm{Re}(z_1)}{\overline{z}_1+\sigma} f_1(-\overline{z}_1) \\ \vdots \\ \frac{2\mathrm{Re}(z_m)}{\overline{z}_m+\sigma} f_m(-\overline{z}_m) \end{bmatrix} \\ &= \frac{1}{f(\sigma)} \sum_{i=1}^{m} \frac{2\mathrm{Re}(z_i)}{(\sigma - \overline{z}_i)(\sigma + \overline{z}_i)} f_i(-\overline{z}_i). \end{aligned}$$

Let us then construct the function

$$g(s) = \frac{f(s)}{s} = \frac{1}{s} \left(\prod_{i=1}^{m} \frac{s - z_i}{s + \overline{z}_i} \right)$$

which in turn can be expanded via partial fraction as

$$g(s) = \sum_{i=1}^{m} \frac{2\mathrm{Re}(z_i) f_i(-\overline{z}_i)}{\overline{z}_i} \frac{1}{s + \overline{z}_i} + f(0)\frac{1}{s}.$$

Since

$$\frac{1}{(s - z_i)(s + z_i)} = \frac{1}{2z_i} \left(\frac{1}{s - z_i} - \frac{1}{s + z_i} \right)$$

it follows that:

$$\begin{aligned} \sum_{i=1}^{m} \frac{2\mathrm{Re}(z_i)}{(\sigma - \overline{z}_i)(\sigma + \overline{z}_i)} f_i(-\overline{z}_i) &= \sum_{i=1}^{m} \frac{2\mathrm{Re}(z_i)}{2\overline{z}_i} f_i(-\overline{z}_i) \times \left(\frac{1}{\sigma - \overline{z}_i} - \frac{1}{\sigma + \overline{z}_i} \right) \\ &= -\frac{1}{2} \left(g(\sigma) + g(-\sigma) \right) \\ &= -\frac{f(\sigma) - f(-\sigma)}{2\sigma}. \end{aligned}$$

Consequently

$$J_2^2(I) = -\frac{1}{f(\sigma)}\frac{f(\sigma)-f(-\sigma)}{2\sigma} = \frac{1}{2\sigma}\left(\frac{1}{|f(\sigma)|^2}-1\right).$$

This completes the proof. ■

For comparison to step tracking, for which $\sigma = 0$ and

$$J_2^2(I) = \sum_{i=1}^{m}\frac{2\mathrm{Re}(z_i)}{|z_i|^2}. \tag{3.12}$$

Theorem 3 is evidently more general and contains additional insight. While a direct analogy here is that the zeros close to $s = \sigma$ can be particularly problematic in tracking the exponential signal, the relative locations of the zeros will also play a more intricate role. For example, in the case of two real zeros, the effect for such configurations as i) z_1, $z_2 < \sigma$; ii) $z_1 < \sigma < z_2$; and iii) $\sigma < z_1$, z_2 will differ. This makes it possible to analyze and to interpret the effect of the so-called "slow" and "fast" zeros relative to the increase of the reference input. Note also that Theorem 3 furnishes a useful connection between the earlier developments and the interpolation approach adopted herein. Indeed, one may observe (3.12) either by taking the limit of (3.10), with $\sigma \to 0$, or by noting that

$$f(0) = \prod_{i=1}^{m}\left(-\frac{\overline{z}_i}{z_i}\right), \quad \overline{f'(0)} = \sum_{i=1}^{m}\frac{2\mathrm{Re}(z_i)}{\overline{z}_i^2}\overline{f_i(z_i)}$$

and henceforth that

$$\begin{aligned} J_2^2(I) =& f(0)\overline{f'(0)} \\ =& \prod_{i=1}^{m}\left(-\frac{\overline{z}_i}{z_i}\right)\left(-\sum_{i=1}^{m}\frac{2\mathrm{Re}(z_i)}{\overline{z}_i^2}\prod_{\substack{j=1\\ j\neq i}}^{m}\left(-\frac{z_j}{\overline{z}_j}\right)\right) \\ =& \sum_{i=1}^{m}\frac{2\mathrm{Re}(z_i)}{|z_i|^2}. \end{aligned}$$

B. Unstable Plants

Tracking in the case of unstable plants based on the one-parameter control structure as given in Fig. 1 is more complex an issue. It is known [5] that in tracking merely a step signal, the plant unstable poles may or may not affect the tracking error. Specifically, while for a SISO system such poles are bound to worsen the tracking performance, for a multivariable system they may only when the input direction is perfectly aligned with one or more pole directions; otherwise, only the nonminimum phase zeros of $P(s)$ will have an effect. This phenomenon can be observed from the present interpolation approach as well. Indeed, it is easy to see that when v is not aligned with any of the pole direction vector, by which we mean that $|\eta_i^{\mathrm{H}} v| \neq 1$ for all $i = 1, \ldots, n$, where η_i are the pole direction vectors, then the transfer function matrix $F(s) - T(s)$ will not be constrained at the poles p_i. Consequently, only the zero interpolation constraints will be in effect, and therefore, in light of Lemma 1, the tracking error will be affected by the plant nonminimum phase zeros only.

When the poles do affect the tracking performance, we may declare at the outset that they worsen it, a fact one can also clearly observe from Lemma 1. Accordingly, the expression of the tracking error becomes substantially more involved, obscuring unfortunately the conceptual insight one desires to obtain. For this reason, we shall focus on a number of simple cases which still lend the insight available. We shall first consider multivariable systems.

Lemma 6: Suppose that $P(s)$ has only one zero $z \in \mathbb{C}_+$ with output zero direction vector w, and one pole $p \in \mathbb{C}_+$ with input pole direction vector v, and that $z \neq p$, $p \neq \sigma$. Then under Assumption 1 and for any stable proper rational $F(s)$

$$\begin{aligned} J_2^2(F) =& \frac{2\mathrm{Re}(z)}{|z-\sigma|^2}\left|w^{\mathrm{H}}F(z)v\right|^2 \\ &+ \frac{2\mathrm{Re}(p)}{|p-\sigma|^2}\left(\|[I-F(p)]v\|^2 - \left|w^{\mathrm{H}}[I-F(p)]v\right|^2\right) \\ &+ \frac{2\mathrm{Re}(p)}{|z-p|^2} \\ &\times\left|\frac{z+\overline{z}}{z-\sigma}w^{\mathrm{H}}F(z)v + \frac{p+\overline{z}}{p-\sigma}w^{\mathrm{H}}[I-F(p)]v\right|^2. \end{aligned} \tag{3.13}$$

Proof: It follows from Lemma 1 by setting $H(s) = [F(s) - T(s)]\widehat{u}(s)$, and noting the interpolation constraints

$$w^{\mathrm{H}}H(z) = \frac{w^{\mathrm{H}}F(z)v}{z-\sigma}, \quad H(p) = \frac{[F(p)-I]v}{p-\sigma}.$$

The proof then follows analogously as in that for Theorem 1. ■

It was shown in [5] that when tracking a step input, the plant unstable poles cannot exert any effect on the tracking performance if the plant is minimum phase, regardless of input directions; this can too be seen from Lemma 1, or Lemma 6. It is clear that for $F(s) = I$, the expression (3.13) becomes

$$\begin{aligned} J_2^2(I) =& \frac{2\mathrm{Re}(z)}{|z-\sigma|^2}\left|w^{\mathrm{H}}v\right|^2\left(1+\frac{4\mathrm{Re}(z)\mathrm{Re}(p)}{|z-p|^2}\right) \\ =& \left|\frac{z+\overline{p}}{z-p}\right|^2\frac{2\mathrm{Re}(z)}{|z-\sigma|^2}\cos^2\angle(w,v) \end{aligned}$$

which was also obtained in [5]. Hence, $J_2(I) = 0$ if $P(s)$ has no zero in $\mathbb{C}_+$. On the other hand, it need not be true for a different $F(s)$. Indeed, for a minimum phase $P(s)$, (3.13) reduces to

$$J_2^2(F) = \frac{2\mathrm{Re}(p)}{|p-\sigma|^2}\|[I-F(p)]v\|^2.$$

In other words, the unstable pole p can actually degrade the tracking performance even for a minimum phase plant, when a different $F(s)$ is used. It is thus expected that while preview control counters the negative effect of plant nonminimum phase zeros, it does so at the expense of worsening the performance degradation due to plant unstable poles, whenever such poles have directions aligned with the input direction. Indeed, in light of Lemma 5 and Lemma 6, the tracking error in this case will become

$$J_2^2(\Lambda) = \frac{2\mathrm{Re}(p)}{|p-\sigma|^2}\sum_{i=1}^{l}\left|1-e^{-T_i p}\right|^2|v_i|^2.$$

The fact can be seen more clearly from the following lower bound of $J_2(\Lambda)$, an immediate consequence of Lemma 5 and Lemma 6.

Corollary 3: Let $\Lambda(s) = e^{-Ts}I$. Suppose that $P(s)$ has only one zero $z \in \mathbb{C}_+$ with output zero direction vector w, and one pole $p \in \mathbb{C}_+$ with input pole direction vector v, and that $z \neq p$, $p \neq \sigma$. Then under Assumption 1,

$$\begin{aligned} J_2^2(\Lambda) \geq& e^{-2\mathrm{Re}(z)T}\frac{2\mathrm{Re}(z)}{|z-\sigma|^2}\cos^2\angle(w,v) \\ &+\left|1-e^{-Tp}\right|^2\frac{2\mathrm{Re}(p)}{|p-\sigma|^2}\sin^2\angle(w,v). \end{aligned} \tag{3.14}$$

Note that to reduce the zero effect mandates to have a large T, but to prevent the effect of the unstable pole requires T to be small, thus exhibiting a conflict between the two requirements. Needless to say, this is always the case for SISO systems.

We end this subsection with a corollary similar to Theorem 3, which gives an exact expression of the tracking error without preview, for

SISO systems. The corollary extends previous results to plants with several unstable poles, whereas elsewhere similar expressions were obtained for plants with a single zero and a single pole only, and in the case of step tracking. The result demonstrates, in the same spirit as (3.13), that the tracking error can become excessively large when the plant has closely located nonminimum phase zeros and poles.

Corollary 4: Let $P(s)$ be a scalar transfer function, and let $\widehat{u}(s)$ be given by (3.1) with $v = 1$. Suppose that $P(s)$ has only one zero $z \in \mathbb{C}_+$ and unstable poles $p_i \in \mathbb{C}_+, z \neq \sigma, z \neq p_i, i = 1, \ldots, n$. Then under Assumption 1

$$J_2^2(I) = \frac{2\mathrm{Re}(z)}{|z-\sigma|^2} \prod_{i=1}^{n} \left| \frac{z+\overline{p}_i}{z-p_i} \right|^2. \tag{3.15}$$

Proof: It proceeds with Lemma 1, by identifying

$$\begin{aligned} X &= 1, \quad Y = \frac{1}{z-\sigma}, \quad V = 0 \\ Q_x &= \frac{1}{2\mathrm{Re}(z)}, \quad Q_{xv} = 0 \\ Q_{yu} &= \frac{1}{z-\sigma} \left[\frac{1}{z-p_1} \quad \cdots \quad \frac{1}{z-p_n} \right] \\ Q_u &= \begin{bmatrix} \frac{1}{p_1+\overline{p}_1} & \cdots & \frac{1}{p_1+\overline{p}_n} \\ \vdots & \cdots & \vdots \\ \frac{1}{p_n+\overline{p}_1} & \cdots & \frac{1}{p_n+\overline{p}_n} \end{bmatrix}. \end{aligned}$$

This leads to

$$\begin{aligned} J_2^2(I) =& \frac{2\mathrm{Re}(z)}{|z-\sigma|^2} + \frac{4\mathrm{Re}(z)\mathrm{Re}(z)}{|z-\sigma|^2} \left[\frac{1}{z-p_1} \quad \cdots \quad \frac{1}{z-p_n} \right] \\ & \times Q_u^{-1} \begin{bmatrix} \frac{1}{\overline{z}-\overline{p}_1} \\ \cdots \\ \frac{1}{\overline{z}-\overline{p}_n} \end{bmatrix}. \end{aligned}$$

Noting the similarity between this expression and (3.11), it can be shown analogously as in the proof for Theorem 3 that

$$\begin{aligned} J_2^2(I) = & \frac{2\mathrm{Re}(z)}{|z-\sigma|^2} + \frac{4\mathrm{Re}(z)\mathrm{Re}(z)}{|z-\sigma|^2} \\ & \frac{1}{2\mathrm{Re}(z)} \left(\prod_{i=1}^{n} = \left| \frac{z+\overline{p}_i}{z-p_i} \right|^2 - 1 \right) \end{aligned}$$

thus completing the proof. ■

IV. Conclusion

This work studies tracking performance limitation problems and extends the previously available results in two aspects. First, it addresses exponentially increasing signals, which are more general than step signals typically studied elsewhere. It was shown that for this class of signals the earlier results can be directly extended, yielding similar conceptual insight and leading to similar conclusions. Specifically, it demonstrates that tracking performance depends on the locations of the plant nonminimum phase zeros relative to the exponent of the reference input, and that it will generally be poor when they are closely located (i.e., when $|z - \sigma|$ is small).

Second, this note examines the use of preview control for tracking. While in the general setting a strong, conceptually appealing result remains unavailable, various bounds on the tracking error were developed, which collectively clarify the role of preview in tracking. It is clear that in general preview is useful for reducing the tracking error resulted from plant nonminimum phase zeros, and indeed it offers one of the few means left for improving tracking performance beyond that provided by causal feedback. Fundamentally, this improvement is made possible by use of the future information of the reference input, and is seen as, unsurprisingly, the advantage of a noncausal tracking scheme over a causal one. Nevertheless, for an unstable plant, the improvement is likely to be compromised by the performance degradation due to the plant unstable poles. It has been found that while it may effectively counter the zero effect, preview actually renders the pole effect worse. Thus, with preview control, there generally exists a conflict between the performance improvement in reducing the zero effect and the further performance degradation due to the plant unstable poles. In light of earlier work on two-parameter tracking scheme, however, preview appears to be a viable strategy when used together with a two-parameter control structure, in which the plant unstable poles do not play any role in tracking.

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