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Robust Stabilization Using Jury Table

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Abstract: Compressed Hankel matrix is given by using orthonormal rational functions constructed from the Jury table. The solutions to the optimal and suboptimal Nehari problems via the compressed Hankel matrix are also given. Robust stabilization problem is reduced to the Nehari problem, so it can also be solved via Jury table.

Keywords: Orthonormal function, Compressed Hankel matrix, Nehari problem, Robust staniliza-tion

1. Introduction

In this paper, we first study Hankel operator and the Nehari problems using the Jury table. After that we reduce the robust stabilization problem to the Nehari problem, so it can also be solved via Jury table.

The motivation is to develop elementary solutions to advanced optimal control problems so to make the advanced optimal control accessible to a wider audience. These new investigation of the connection between advanced optimal and robust control problems and the classical tools yield the solution to the Nehari problems. Since the problem plays a fundamental role in H_∞ optimal control theory, its elementary solution opens the door for a simple, polynomial approach to H_∞ optimal control theory. Similar study for continuous time systems is also carried out by Qiu [13].

2. Jury Table and Orthonormal Functions

Consider a stable polynomial

$$a(z) = a_0 z^n + a_1 z^{n-1} + \cdots + a_n,$$

where $a_i \in \mathbb{R}$ and $a_0 > 0$. It is said to be stable if all of its roots are inside the unit disk.

Construct the Jury table [10] as in Table 1. In the Jury table, the first row is copied from the coefficients of the polynomial,

$$r_{00} = a_0,\ r_{01} = a_1,\ \ldots,\ r_{0n} = a_n. \tag{1}$$

The row r_i^*, $i = 0, \cdots, n-1$, is obtained by writing the elements of the preceding row in the reverse order. The row r_{i+1}, $i = 0, \cdots, n-1$, is computed from its two preceding rows r_{i-1} and r_{i-1}^* as

$$r_{(i+1)j} = \frac{1}{r_{i0}} \begin{vmatrix} r_{ij} & r_{i(n-i)} \\ r_{i(n-i-j)} & r_{i0} \end{vmatrix}, \tag{2}$$

for $i = 0, \ldots, n-1,\ j = 0, \ldots, n-i-1$.

Table 1: Jury Table

r_0	r_{00}	r_{01}	$\cdots$	$r_{0(n-1)}$	r_{0n}
r_0^*	r_{0n}	$r_{0(n-1)}$	$\cdots$	r_{01}	r_{00}
r_1	r_{10}	r_{11}	$\cdots$	$r_{1(n-1)}$	
r_1^*	$r_{1(n-1)}$	$r_{1(n-2)}$	$\cdots$	r_{10}	
$\vdots$	$\vdots$				
r_{n-1}	$r_{(n-1)0}$	$r_{(n-1)1}$			
r_{n-1}^*	$r_{(n-1)1}$	$r_{(n-1)0}$			
r_n	r_{n0}				

The Jury stability criterion states that $a(z)$ is stable if and only if $r_{i0} > 0$ for all $i = 1, \ldots, n$.

Consider the set of strictly proper rational functions with denominator $a(z)$

$$\mathcal{X}_a = \left\{ \frac{b(z)}{a(z)},\ \deg b(z) < \deg a(z) \right\}. \tag{3}$$

Clearly, $\mathcal{X}_a$ is an n-dimensional subspace of $\mathcal{RH}_2$. In applications, as evidenced later in this paper, it is desirable to find a basis, or better an orthonormal basis of $\mathcal{X}_a$.

The Jury table can be used to construct such an orthonormal basis of $\mathcal{X}_a$, see [2], [4] and [17]. Recall the Jury table of $a(z)$ and for the rows r_i, $i = 1, 2, \ldots, n$, define polynomials

$$\begin{aligned} r_1(z) &= r_{10} z^{n-1} + r_{11} z^{n-2} + \cdots + r_{1(n-1)} \\ &\vdots \\ r_{n-1}(z) &= r_{(n-1)0} z + r_{(n-1)1} \\ r_n(z) &= r_{n0}. \end{aligned} \tag{4}$$

Since $a(z)$ is stable, $r_{i0} > 0$, for $i = 1, 2, \ldots, n$. We can define

$$\alpha_i = \sqrt{\frac{r_{00}}{r_{i0}}},\ i = 0, 1, 2, \ldots, n.$$

Theorem 1 *The functions* $E_i(z) = \alpha_i \frac{r_i(z)}{a(z)}$, $i = 1, 2, \ldots, n$, *form orthonormal basis of* $\mathcal{X}_a$.

3. Hankel Operator and Compressed Hankel Matrix

Hankel operators find various applications in engineering problems such as in model reduction and optimal control. The computation of the Hankel singular values and Schmidt pairs is the key for these applications and is studied in [1), 9)] and [7)]. Let $P_+ : \mathcal{L}_2 \to \mathcal{H}_2$ and $P_- : \mathcal{L}_2 \to \mathcal{H}_2^\perp$ denote the orthogonal projections such that

$$P_+\left(\sum_{k=-\infty}^{\infty} f(k)z^{-k}\right) = \sum_{k=0}^{\infty} f(k)z^{-k},$$
$$P_-\left(\sum_{k=-\infty}^{\infty} f(k)z^{-k}\right) = \sum_{k=-\infty}^{-1} f(k)z^{-k}.$$

Let $J : \mathcal{L}_2 \to \mathcal{L}_2$ denote the reversal operator and $S : \mathcal{L}_2 \to \mathcal{L}_2$ denote the backward shift operator such that

$$JF(z) = F(z^{-1}),\ SF(z) = zF(z).$$

Definition *Given a stable system with strictly proper transfer function $G(z)$, the associated Hankel operator $\Gamma_G : \mathcal{H}_2^\perp \to \mathcal{H}_2$ is defined by*

$$\Gamma_G U(z) = P_+(G(z)U(z)),\ U(z) \in \mathcal{H}_2^\perp.$$

It is well-known that Γ_G is a finite rank operator when $G(z)$ is rational.

Lemma 1 [7)] *Let $G(z) = \dfrac{b(z)}{a(z)}$ be a strictly proper stable transfer function. Then*

$$\text{Im }\Gamma_G = S\mathcal{X}_a,\ (\text{Ker }\Gamma_G)^\perp = J\mathcal{X}_a.$$

The Hankel operator Γ_G is the orthogonal direct sum of a zero operator and a compression of Γ_G mapping $J\mathcal{X}_a$ into $S\mathcal{X}_a$. Everything interesting about it is contained in the compression.

This compressed Hankel operator can be represented by a matrix if we choose a basis in $(\text{Ker}H_G)^\perp$ and a basis in $\text{Im}H_G$. Note that both $(\text{Ker}H_G)^\perp$ and $\text{Im}H_G$ are isomorphic to $\mathcal{X}_a$. Hence we can use the orthonormal basis of $\mathcal{X}_a$

$$E(z) := [\ E_1(z) \quad E_2(z) \quad \cdots \quad E_n(z)\ ]$$

defined in Theorem 1 to form an orthonormal basis in $(\text{Ker}H_G)^\perp$

$$E(z^{-1}) = [\ E_1(z^{-1}) \quad E_2(z^{-1}) \quad \ldots \quad E_n(z^{-1})\ ]$$

and one in $\text{Im}H_G$

$$zE(z) = [\ zE_1(z) \quad zE_2(z) \quad \ldots \quad zE_n(z)\ ].$$

We call the matrix representation under this basis *Compressed Hankel Matrix* and denote it by H_G. The singular values of H_G are the Hankel singular values of $G(z)$ and are denoted by $\sigma_1, \sigma_2, \ldots, \sigma_n$. We assume that $\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_n$. The largest singular value is the Hankel norm of $G(z)$ and is denoted by $\|G(z)\|_H$. Let (u_i, v_i) be a left and right singular vectors of H_G corresponding to σ_i and let

$$U_i(z) = E(z^{-1})u_i,\ V_i(z) = zE(z)v_i.$$

Then $(U_i(z), V_i(z))$ is a Schmidt pair of Γ_G corresponding to σ_i.

We are interested in computing the Hankel singular values and Schmidt pairs of Γ_G, the key is to find H_G from $G(z) = \dfrac{b(z)}{a(z)}$. The following result can be found in [17)].

Theorem 2 *Construct the Jury table of $a(z)$. Define matrix A as in (7) and M as:*

$$M = \begin{bmatrix} \alpha_1 r_{10} & 0 & \cdots & 0 \\ \alpha_1 r_{11} & \alpha_2 r_{20} & \ddots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ \alpha_1 r_{1(n-1)} & \alpha_2 r_{2(n-2)} & \cdots & \alpha_n r_{n0} \end{bmatrix}.$$

where $k_i = \frac{r_{i(n-i)}}{r_{i0}}$, $i = 0, 1, \ldots, n$. Then

$$H_G = a^{\sim}(A)^{-1} b(A) M^{-1} \begin{bmatrix} 0 & \cdots & 1 \\ \vdots & \cdot^{\cdot^{\cdot}} & \vdots \\ 1 & \cdots & 0 \end{bmatrix} M. \quad (5)$$

The adjoint Hankel operator $\Gamma_G^* : \mathcal{H}_2 \to \mathcal{H}_2^\perp$ is given by

$$\Gamma_G^* U(z) = P_-(G(z^{-1})U(z)),\ U(z) \in \mathcal{H}_2$$

and

$$\text{Im}\Gamma_G^* = J\mathcal{X}_a,\ (\text{Ker}\Gamma_G^*)^\perp = S\mathcal{X}_a.$$

Corollary 1 *The adjoint Hankel operator Γ_G^* satisfies*

$$\Gamma_G^* = SJ\Gamma_G SJ. \quad (6)$$

$$A = \begin{bmatrix} -k_0k_1 & \alpha_1/\alpha_2 & \cdots & 0 & 0 \\ -k_0k_2\alpha_1/\alpha_2 & -k_1k_2 & \ddots & 0 & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ -k_0k_{n-1}\alpha_1/\alpha_{n-1} & -k_1k_{n-1}\alpha_2/\alpha_{n-1} & \cdots & -k_{n-2}k_{n-1} & \alpha_{n-1}/\alpha_n \\ -k_0k_n\alpha_1/\alpha_n & -k_1k_n\alpha_2/\alpha_n & \cdots & -k_{n-2}k_n\alpha_{n-1}/\alpha_n & -k_{n-1}k_n \end{bmatrix}. \quad (7)$$

Remark 1 : Corollary 1 implies that the compressed matrix representation of Γ_G^* is also H_G. By definition, the matrix representation of Γ_G^* is H_G'. Hence H_G must be symmetric and

$$U_i(z) = \pm z V_i(z^{-1}) = \pm SJV_i(z). \quad (8)$$

This fact may offer some simplification in the computation.

4. Solution to Nehari Problem

In this section, we apply the materials in the last section to the solutions of the optimal and suboptimal Nehari problem. The Nehari problem [12] plays an important role in robust and optimal control, it is an approximation problem with respect to the $\mathcal{L}_\infty$ norm: Given a stable strictly proper system $G(z) = \frac{b(z)}{a(z)}$, find $Q(z) \in \mathcal{H}_\infty$ to minimize $\|G(z^{-1}) - Q(z)\|_\infty$. The following theorem is well-known [1], see also [7].

Theorem 3 *Let $(U_1(z), V_1(z))$ be the Schmidt pair of H_G corresponding to the largest Hankel singular value σ_1. Then*

$$\min_{Q(z)\in\mathcal{H}_\infty} \|G(z^{-1}) - Q(z)\|_\infty = \sigma_1,$$

and the unique minimizing $Q(z)$ is given by

$$Q(z) = G(z^{-1}) - \sigma_1 \frac{V_1(z^{-1})}{U_1(z^{-1})}.$$

Since the Hankel singular values and Schmidt pairs can be obtained using the orthonormal basis constructed from the Jury table, a computational method for solving the Nehari problem is thus obtained.

The suboptimal Nehari problem is to characterize all $Q(z) \in \mathcal{H}_\infty$ such that $\|G(z^{-1}) - Q(z)\|_\infty \le \gamma$ with $\|G(z)\|_H < \gamma$. It is studied in [6], [3] and [8], the methods in these papers are all related to the state space system theory. Our approach to the solution will be based on the orthonormal basis and the compressed Hankel matrix H_G in Theorem 2.

We also define the entropy of $F(z)$ as

$$\mathcal{I}[F(z)] = -\frac{\gamma^2}{2\pi}\int_{-\pi}^{\pi} \ln[1 - \gamma^{-2}F(e^{-j\omega})F(e^{j\omega})]d\omega.$$

Given a strictly proper transfer function $G(z) = \frac{b(z)}{a(z)}$, we can expand $G(z)$ as

$$G(z) = \beta_1 E_1(z) + \ldots + \beta_n E_n(z) = E(z)\beta, \quad (9)$$

where $E(z) = [\ E_1(z)\quad E_2(z)\quad \cdots\quad E_n(z)\ ]$ are the orthonormal functions constructed from Jury table.

Finding $\beta_i, i = 1, \ldots, n$, is simple. One only need to compare the coefficients in (9) and solve a set of linear equations. It turns out that these equations have special structure and we can obtain the orthonormal basis and these coefficients β_i simultaneously by using the augmented Jury table, see details in [17].

Theorem 4 *Let $G(z) = \frac{b(z)}{a(z)} \in \mathcal{H}_\infty$ be rational, strictly proper and $\|G(z)\|_H < \gamma$. Expand $G(z)$ as*

$$G(z) = E(z)\beta$$

and let

$$\alpha = \sqrt{1 + \beta'(\gamma^2 I - H_G^2)^{-1}\beta} \quad (10)$$

$$X(z) = \gamma E(z)(\gamma^2 I - H_G^2)^{-1}\beta/\alpha \quad (11)$$

$$Y(z) = [1 + zE(z)H_G(\gamma^2 I - H_G^2)^{-1}\beta]/\alpha. \quad (12)$$

(1) Define

$$V(z) = \begin{bmatrix} V_{11}(z) & V_{12}(z) \\ V_{21}(z) & V_{22}(z) \end{bmatrix}, \quad (13)$$

where

$$\begin{aligned} V_{11}(z) &= Y(z^{-1}) - \gamma^{-1}G(z^{-1})X(z) \\ V_{12}(z) &= X(z^{-1}) - \gamma^{-1}G(z^{-1})Y(z) \\ V_{21}(z) &= X(z) \\ V_{22}(z) &= Y(z). \end{aligned} \quad (14)$$

Then the set of all $Q(z)$ such that $\|G(z^{-1}) - Q(z)\|_\infty \le \gamma$ is given by

$$\{Q(z) = -\gamma\mathcal{L}[V(z), R(z)] : R(z) \in \mathcal{H}_\infty, \|R(z)\|_\infty \le 1\},$$

where

$$\mathcal{L}[V(z), R(z)] = \frac{V_{11}(z)R(z) + V_{12}(z)}{V_{21}(z)R(z) + V_{22}(z)}.$$

(2) Define

$$P(z) = \begin{bmatrix} P_{11}(z) & P_{12}(z) \\ P_{21}(z) & P_{22}(z) \end{bmatrix} = \frac{1}{Y(z)}\begin{bmatrix} U(z) & 1 \\ 1 & -X(z) \end{bmatrix}, \quad (15)$$

with

$$U(z) = X(z^{-1}) - \gamma^{-1}G(z^{-1})Y(z). \quad (16)$$

Then the set of all $Q(z)$ such that $\|G(z^{-1}) - Q(z)\|_\infty \le \gamma$ is given by

$$\{Q(z) = -\gamma\mathcal{F}[P(z), R(z)], R(z) \in \mathcal{H}_\infty, \|R(z)\|_\infty \le 1\}$$

where

$$\begin{aligned} &\mathcal{F}[P(z), R(z)] \\ &= P_{11}(z) + P_{12}(z)R(z)(I - P_{22}(z)R(z))^{-1}P_{21}(z). \end{aligned}$$

(3) By setting $R(z) = 0$, the central $Q(z)$ satisfying $\|G(z^{-1}) - Q(z)\|_\infty \le \gamma$ which minimizes $\mathcal{I}[G(z^{-1}) - Q(z)]$ is given by

$$Q(z) = -\gamma V_{12}(z)V_{22}^{-1}(z) = -\gamma P_{11}(z)$$

and

$$G(z^{-1}) - Q(z) = \gamma \frac{X(z^{-1})}{Y(z)}.$$

Example 1

For

$$G(z) = \frac{b(z)}{a(z)} = \frac{\sqrt{2}z + 0.5}{z^2 + \sqrt{2}z + 0.5},$$

we wish to find all $Q(z) \in \mathcal{H}_\infty$ such that $\|G(z^{-1}) - Q(z)\|_\infty \le \gamma$ with $\gamma = 8$.

Construct the Jury table, we can get

$$\alpha_0 = 1,\ \alpha_1 = \frac{2\sqrt{3}}{3},\ \alpha_2 = 2\sqrt{3}$$

$$k_0 = 0.5,\ k_1 = \frac{2\sqrt{2}}{3},\ k_2 = 1$$

$$E_1(z) = \frac{\sqrt{3}/2z + \sqrt{6}/3z}{z^2 + \sqrt{2} + 0.5},\ E_2(z) = \frac{\sqrt{3}/6}{z^2 + \sqrt{2} + 0.5},$$

and

$$\beta = \begin{bmatrix} \frac{2\sqrt{6}}{3} & \frac{-5\sqrt{3}}{3} \end{bmatrix}'.$$

Hence,

$$A = \begin{bmatrix} -\frac{\sqrt{2}}{3} & \frac{1}{3} \\ -\frac{1}{6} & -\frac{2\sqrt{2}}{3} \end{bmatrix},\ M = \begin{bmatrix} \frac{\sqrt{3}}{2} & 0 \\ \frac{\sqrt{6}}{3} & \frac{1}{\sqrt{12}} \end{bmatrix}.$$

and

$$H_G = \begin{bmatrix} 1.8856 & -3.3333 \\ -3.3333 & 3.7712 \end{bmatrix}, \sigma_1 = 6.2925.$$

Now let

$$X(z) = \frac{0.43z + 0.2}{z^2 + \sqrt{2}z + 0.5}$$
$$Y(z) = \frac{1.2z^2 + 1.37z + 0.42}{z^2 + \sqrt{2}z + 0.5}.$$

We can get

$$V(z) = \begin{pmatrix} \frac{0.83z^2 + 1.50z + 0.60}{z^2 + \sqrt{2}z + 0.5} & \frac{0.24z^2 + 0.14z}{z^2 + \sqrt{2}z + 0.5} \\ \frac{0.43z + 0.20}{z^2 + \sqrt{2}z + 0.5} & \frac{1.20z^2 + 1.37z + 0.42}{z^2 + \sqrt{2}z + 0.5} \end{pmatrix},$$

$$P(z) = \begin{pmatrix} \frac{0.24z^2 + 0.14z}{1.20z^2 + 1.37z + 0.42} & \frac{z^2 + \sqrt{2}z + 0.5}{1.20z^2 + 1.37z + 0.42} \\ \frac{z^2 + \sqrt{2}z + 0.5}{1.20z^2 + 1.37z + 0.42} & -\frac{0.43z + 0.20}{1.20z^2 + 1.37z + 0.42} \end{pmatrix}.$$

By setting $R(z) = 0$, the unique $Q(z)$ satisfying $\|G(z^{-1}) - Q(z)\|_\infty \le 8$ which minimizes $\mathcal{I}[G(z^{-1}) - Q(z)]$ is given by

$$Q(z) = -8\frac{0.24z^2 + 0.14z}{1.20z^2 + 1.37z + 0.42}.$$

5. Robust Stabilization

In this section, we will study a typical robust stabilization problem [18), 14)]. In this problem, we design a controller K, for a given plant P, such that the following quantity is maximized.

$$b_{P,K} := \left\| \begin{bmatrix} I \\ K \end{bmatrix} (I + PK)^{-1} [\ I \quad P\ ] \right\|_\infty^{-1}.$$

This quantity gives a measure of the robustness of the feedback system under the gap metric or ν-gap metric uncertainty. Hence the robust stabilization problem is a special discrete-time $\mathcal{H}_\infty$ optimal control problem.

$$\inf_{K \text{ stabilizing}} \left\| \begin{bmatrix} I \\ K \end{bmatrix} (I + PK)^{-1} [\ I \quad P\ ] \right\|_\infty.$$

We are interested in finding suboptimal controllers.

Let us first recall the Youla parameterization of all stabilizing controllers. For a proper system $P(z) = \frac{b(z)}{a(z)}$ where $a(z)$ and $b(z)$ are coprime polynomials with degree n. We first find the spectral factor $d(z)$ such that

$$z^n[a(z)a(z^{-1}) + b(z)b(z^{-1})] = z^n d(z)d(z^{-1}).$$

Then, we solve the following Doiphantine equation

$$a(z)x(z) + b(z)y(z) = d^2(z).$$

Define

$$M(z) = \frac{a(z)}{d(z)},\ N(z) = \frac{b(z)}{d(z)}$$
$$\tilde{M}(z) = \frac{y(z)}{d(z)},\ \tilde{N}(z) = \frac{x(z)}{d(z)}.$$

Then the set of all controller $K(z)$ that internally stabilize $P(z)$ is given by

$$K(z) = \frac{\tilde{M}(z) - M(z)Q(z)}{\tilde{N}(z) + N(z)Q(z)} \tag{18}$$

for $Q(z) \in \mathcal{RH}_\infty$. Apply the parameterized controller to the above $\mathcal{H}_\infty$ problem, we can get

$$\begin{aligned} &\left\| \begin{bmatrix} I \\ K \end{bmatrix} (I + PK)^{-1} [\ I \quad P\ ] \right\|_\infty \\ &= \left\| \begin{bmatrix} \frac{x(z)b(z^{-1}) - y(z)a(z^{-1})}{d(z)d(z^{-1})} + Q(z) \\ 1 \end{bmatrix} \right\|_\infty \\ &= \sqrt{\left\| \frac{y(z)a(z^{-1}) - x(z)b(z^{-1})}{d(z)d(z^{-1})} - Q(z) \right\|_\infty + 1} \\ &= \sqrt{\left\| \frac{w(z^{-1})}{d(z^{-1})} - Q(z) \right\|_\infty + 1} \end{aligned}$$

for some polynomial $w(z)$ that satisfies

$$z^n[y(z)a(z^{-1}) - x(z)b(z^{-1})] = z^n d(z)w(z^{-1}). \tag{19}$$

Let $G(z) = \frac{w(z)}{d(z)}$, the original robust stabilization problem reduce to find $Q(z) \in \mathcal{RH}_\infty$ to minimize

$$\left\|G(z^{-1}) - Q(z)\right\|_\infty . \tag{20}$$

Let

$$G(z) = G_s(z) + G(\infty)$$

where $G_s(z)$ is a strictly proper transfer function. Also let $Q_1(z) = Q(z) - G(\infty)$, then equation (20) becomes

$$\left\|G_s(z^{-1}) - Q_1(z)\right\|_\infty$$

which is a Nehari problem solved in Section 4.

Example 2 Consider

$$P(z) = \frac{1.5}{z^2+1}.$$

We wish to find the the suboptimal controller $K(s)$.
Step 1: (Spectral factorization) From

$$(z^2+1)(z^2+1) + 1.5^2 z^2 = z^2 d(z) d(z^{-1}),$$

we can get

$$d(z) = 2z^2 + 0.5.$$

Step 2 (Diophantion equation) From

$$(z^2+1)x(z) + 1.5y(z) = (2z^2+0.5)^2,$$

we can get

$$x(z) = 4z^2 - 2,\ y(z) = 1.5.$$

From

$$(1.5(z^2+1) - 4z^2 - 2)1.5z^2 = (2z^2+0.5)(3z^2-3),$$

we get

$$w(z) = 3z^2 - 3.$$

Hence

$$G(z) = \frac{3z^2-3}{2z^2+0.5}.$$

Step 3 (Suboptimal Nehari problem) Let

$$G_s(z) = G(z) - G(\infty) = \frac{-3.75}{2z^2+0.5}, \gamma = 3.$$

Solve

$$\|G_s(z^{-1}) - Q_1(z)\|_\infty < 3, , Q_1(z) \in \mathcal{RH}_\infty.$$

We get

$$V(z) = \begin{bmatrix} \frac{1.069z^2+0.4677}{2z^2+0.5} & \frac{0.1336z^2}{2z^2+0.5} \\ \frac{1.203}{2z^2+0.5} & \frac{1.871z^2+0.2673}{2z^2+0.5} \end{bmatrix}.$$

Hence, the central solution such that

$$\|G(z^{-1}) - Q(z)\|_\infty < 3, , Q(z) \in \mathcal{RH}_\infty$$

is given by

$$Q(z) = 1.5 + \frac{0.1336z^2}{1.871z^2+0.2673} = \frac{2.94z^2+0.4009}{1.871z^2+0.2673}.$$

The central controller K is given by

$$\begin{aligned} K &= \frac{1.5(1.871z^2+0.2673) - (z^2+1)(2.94z^2+0.4009)}{(4z^2-2)(1.871z^2+0.2673) + 1.5(2.94z^2+0.4009)} \\ &= \frac{-2.94z^4}{7.848z^4+1.737z^2} \\ &= -\frac{z^2}{2.55z^2+0.59}. \end{aligned}$$

6. Conclusion

Compressed Hankel matrix is given by using orthonormal rational functions constructed from the Jury table. The solutions to the optimal and suboptimal Nehari problems via the compressed Hankel matrix are also given. Robust stabilization problem is reduced to the Nehari problem, so it can also be solved via Jury table.

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