2008 10th Intl. Conf. on Control, Automation, Robotics and Vision
Hanoi, Vietnam, 17–20 December 2008

Stabilization of Networked Multi-Input Systems with Channel Resource Allocation

Guoxiang Gu
Department of Electrical and Computer Engineering
Louisiana State University
Baton Rouge, LA 70803-5901 USA
ggu@lsu.edu

Li Qiu
Department of Electronic and Computer Engineering
Hong Kong University of Science and Technology
Clear Water Bay, Kowloon, Hong Kong
eeqiu@ust.hk

***Abstract*—In this paper, we study the problem of stabilizing a linear time-invariant discrete-time system with information constraints in the input channels. The information constraint in each input channel is modeled as a sector uncertainty. Equivalently, the transmission error of an input channel is modeled as an additive system uncertainty with a bound in the induced norm. We attempt to find the least information required, or equivalently the largest allowable uncertainty bound, in each input channel which renders the stabilization possible. The solution for the single-input case, which gives a typical $\mathcal{H}_\infty$ optimal control problem, is available in the literature and is given analytically in terms of the Mahler measure or topological entropy of the plant. The main purpose of this paper is to address the multi-input case. In the multi-input case, if the information constraint in each input channel is given *a priori*, then our stabilization problem turns out to be a so-called μ synthesis problem, a notoriously hard problem. In this paper, we assume that the information constraints in the input channels are determined by the network resources assigned to the channels and they can be allocated subject to a total recourse constraint. With this assumption, the resource allocation becomes part of the design problem and a modified μ synthesis problem arises. Surprisingly, this modified μ-synthesis problem can be solved analytically and the solution is also given in terms of the Mahler measure or topological entropy as in the single-input case.**

***Index Terms*—Networked control, networked stabilization, Mahler measure, topological entropy, channel resource allocation.**

I. Introduction

Networked feedback control systems have attracted great attention recently. See the special issues [2], [3], as well as many papers on this topic in recent control journals and conferences. See also the survey papers [10], [14]. One fundamental issue studied in the context of networked control is stabilization with information constraint in the input channel. The information constraint in the input channel takes various forms in different studies, such as data-rate constraint [4], [15], quantization [8], [9], signal-to-noise ratio (SNR) constraint [6], etc. In [9], logarithmic quantizers are used in the input channels and are considered as sector uncertainties. $\mathcal{H}_\infty$-based robust control technique is used to design the stabilization controller. A nice analytic solution is obtained for the coarsest quantization, corresponding to the minimum amount of information, required to accomplish stabilization in the single-input case. The analytic solution is given in terms of the Mahler measure of the plant, i.e., the absolute product of the unstable poles. Motivated from this, we in this paper model the information constraint in an input channel as a general sector uncertainty, not necessarily a quantizer, not necessarily memoryless, not necessarily time-invariant. In other words, instead of studying only the logarithmic quantizers considered as sector uncertainties, we study sector uncertainties covering logarithmic quantizers as special cases. This gives the potential to use sector uncertainty to model other network features such as packet drops and transmission delays. Equivalently, we model the information constraint as a transmission error considered as a uncertain system with a norm bound. We also put our emphasis in multi-input systems where each input channel has an information constraint, i.e., a transmission error system with an induced norm bound. $\mathcal{H}_\infty$-based robust control is then used to determine the least information, or largest error bounds, under which the stabilization is possible. Since there are multiple uncertainties in the multiple input channels, this problem, on surface, appears like a μ-synthesis problem. However, we will introduce a new twist. Instead of having the information constraints or uncertainty bounds in the input channels specified *a priori* as in μ-synthesis, we assume that they can be allocated by the controller designer subject a total resource constraint. With this new twist, rather surprisingly, the problem becomes analytically solvable and the solution is given in terms of the Mahler measure or topological entropy of the plant as in various studies in [4], [15], [8], [9], [6] for signal-input systems.

II. Problem Formulation

Consider a discrete-time system described by state-space equation

$$x(k+1) = Ax(k) + Bu(k) \tag{1}$$

where $u(k) \in \mathbb{R}^m$ and $x(k) \in \mathbb{R}^n$. We will denote this system by $[A|B]$ for simplicity. Assume that $[A|B]$ is stabilizable and that the state variable $x(k)$ is available for feedback control. We are interested in stabilizing the system by a constant state

978-1-4244-2287-6/08/$25.00

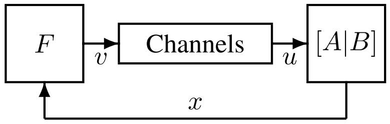

Fig. 1. State feedback via transmission channels

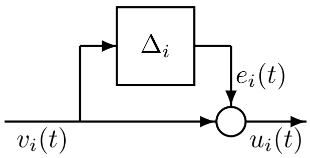

Fig. 2. A transmission channel

feedback. What is different from the standard setup studied 40 years ago as for example in [18] is that now in the network era the signal transmission from each element in the controller output vector to the corresponding element in the plant input vector is via a communication channel. The new setup is shown in Figure 1.

How a communication channel, especially one in feedback control, should be modelled is a big issue. There is a vast literature on this and different channel modelling gives rise to a different control method. In this paper, motivated by [9], we model a channel as an ideal transmission system with a unity transfer function together with an additive norm bounded uncertainty, as shown in Figure 2. The uncertainty Δ_i can be a nonlinear, time-varying, and dynamic system. We only assume that its ℓ_2-induced norm is bounded by δ_i, i.e., $\|\Delta_i\| \leq \delta_i$. We intend to use this uncertainty to model the possible transmission errors due to quantization, network delay, signal distortion, and packet drops, as well as other inherent uncertainty in the plant input due to actuator inaccuracy. In other words, we measure the information limitation in an input channel by the uncertainty norm bound δ_i. Larger δ_i corresponds to less reliable information being transmitted through the channel. The inverse δ_i^{-1} of the norm bound can be considered as the worst case signal-to-error ratio (SER) since

$$\|\Delta_i\|^{-1} = \inf_{v_i(t)\in\ell_2} \frac{\|v_i(t)\|_2}{\|e_i(t)\|_2} \geq \delta_i^{-1}$$

and it can be used to measure the channel accuracy, capacity and reliability. Here we use transmission error and SER instead of transmission noise and SNR as in the communication theory to distinguish the difference of channels in closed-loop and open-loop applications.

One strong motivation for this channel model is the use of the logarithmic quantizer advocated in [8]. A logarithmic quantizer, depicted in Figure 3, is defined by the following

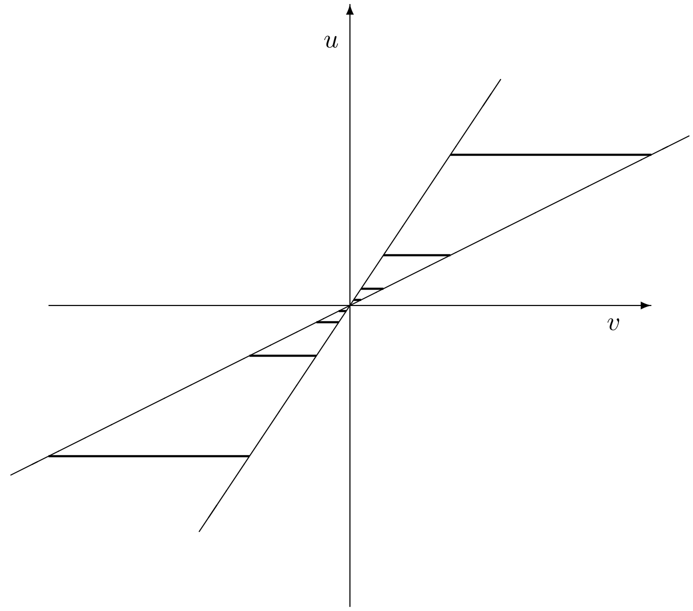

Fig. 3. A logarithmic quantizer

nonlinear mapping:

$$u = Q_\delta(v) := \begin{cases} \rho^l u_0, & \text{if } \frac{\rho^l u_0}{1+\delta} < v \leq \frac{\rho^l u_0}{1-\delta} \\ 0, & \text{if } v = 0 \\ -Q_\delta(-v), & \text{if } v < 0 \end{cases} \tag{2}$$

where $u_0 > 0$, $0 < \rho < 1$, $\delta = \frac{1-\rho}{1+\rho}$, and $l = 0, \pm 1, \pm 2, \ldots$.

For such a quantizer, the quantization error has a norm bound

$$\frac{\|v(k) - u(k)\|_2}{\|v(k)\|_2} \leq \delta.$$

Apparently, a quantizer belongs to the channel model mentioned above.

We are interested in finding smallest accuracy in the channels such that the state feedback stabilization is possible, i.e., we are interested in finding the largest possible $d_1, d_2, \ldots, d_m$ such that the feedback gain F can be designed so that the closed loop system is stable. When applied to the logarithmic quantizer case, this corresponds to finding the coarsest quantizers so that the state feedback stabilization is possible. When there are several input channels, what one means by largest error bounds or coarsest quantizers needs clarification and will becomes precise in the following.

Before preceding, let us recall two concepts which was introduced in dynamic system theory long time ago but only appeared in control literature very recently. One is the Mahler measure [13] of an $n \times n$ matrix A, denoted by $M(A)$, which is simply the absolute value of the product of the unstable eigenvalues of A, i.e.,

$$M(A) = \prod_{i=1}^{n} \max\{1, |\lambda_i(A)|\}.$$

The second is the topological entropy [1] of A, denoted by $h(A)$, which is simply the logarithm of $M(A)$, i.e., $h(A) = \log M(A)$.

In the single-input case, we have $m = 1$. Since there is only one transmission channel, we drop the subscript in Δ_1 and d_1. We ask what is the largest δ so that the networked control system can be stabilized by designing a feedback gain F for all possible uncertainty satisfying the norm bound. This is a typical $\mathcal{H}_\infty$ robust control problem. Starting from the analysis problem, for a fixed norm bound δ and a fixed stabilizing feedback gain F, it follows from the small gain theorem that the uncertain system is robustly stable if and only if

$$\delta < \|T(z)\|_\infty^{-1}$$

where $T(z)$ is the so-called complementary sensitivity function of the feedback system given by

$$T(z) = F(zI - A - BF)^{-1}B.$$

The largest δ we need is then given by solving an optimal synthesis problem: given $[A|B]$, find

$$\inf_{F:A+BF \text{ is stable}} \|T(z)\|_\infty.$$

The following theorem is shown in [9].

Theorem 2.1: It holds that

$$\inf_{F:A+BF \text{ is stable}} \|T(z)\|_\infty = M(A).$$

Namely, the uncertain system can be robustly stabilized by designing F if and only if $\delta < M(A)^{-1}$.

For multi-input systems, the problem is more complicated. Since there are more than one uncertainties in the loop, the robust stability and stabilization problems are called structured problems. If the uncertainty bounds $\delta_1, \delta_2, \ldots, \delta_m$ are given by the God and is absolutely untouchable, and if a stabilizing feedback gain F is also given, the uncertain system is stabilized if and only if [16]

$$\inf_{D \in \mathcal{D}} \|D^{-1}T(z)DD_\delta\|_\infty < 1 \tag{3}$$

where $D_\delta = \operatorname{diag}(\delta_1, \delta_2, \ldots, \delta_m)$ and $\mathcal{D}$ is the set of all $m \times m$ diagonal matrices with positive diagonal entries. The minimization problem in (3) is convex hence is manageable. However the design problem, which is to find a stabilizing F such that (3) holds, is notoriously hard. This design problem is more or less equivalent to the minimization problem

$$\inf_{F:A+BF \text{ is stable}} \left[\inf_{D \in \mathcal{D}} \|D^{-1}T(z)DD_\delta\|_\infty\right]. \tag{4}$$

The objective function here is convex over D and also convex over F but is not jointly convex.

In networked control, very often the SER δ_i is (inversely) associated with certain resource. If we allocate more resource to the i-th channel, then we are able to reduce the SER. For example, the use of better and expensive hardware in the i-th channel may reduce δ_i; allocate more communication bandwidth to the i-th channel may also reduce δ_i. Then in the networked control problem, we might have an overall constraint in the resource but we do have the freedom to allocate the resource among different channels. Let us assume that the overall resource constraint is given in terms of $\delta = \prod_{i=1}^m \delta_i$. Then the controller designer is in the position to allocate δ_i optimally among the channels, so that the expression (4) is minimize. This gives rise to a further nested minimization problem: given $[A|B]$ and $d > 0$, find

$$\inf_{\det D_\delta = \delta} \left\{ \inf_{F:A+BF \text{ is stable}} \left[\inf_{D \in \mathcal{D}} \|D^{-1}T(z)DD_\delta\|_\infty\right]\right\}.$$

This problem looks even harder than (4), but rather surprisingly it admits a very nice analytic solution, which will be given in the next section.

III. Main Result

We first state the solution to a special discrete-time state feedback $\mathcal{H}_\infty$ control problem in which the objective function is a weighted complimentary sensitivity function [5], [11].

Lemma 3.1: Assume that $[A|B]$ is stabilizable. Then there exists a stabilizing state feedback gain F such that $\|F(zI - A - BF)^{-1}BW\|_\infty < 1$ if and only if there exists a stabilizing solution $X \geq 0$ to Riccati equation

$$A'X\left[I + B(I - WW')B'X\right]^{-1}A = X \tag{5}$$

satisfying $I - W'B'XBW > 0$. If such an $X \geq 0$ exists, then a desired F is given by

$$F = -\left[I + B'XB(I - WW')\right]^{-1}B'XA.$$

The main result in this paper is presented in the next theorem.

Theorem 3.2: Inequality

$$\inf_{\det D_\delta = \delta} \left\{ \inf_{F:A+BF \text{ is stable}} \left[\inf_{D \in \mathcal{D}} \|D^{-1}T(z)DD_\delta\|_\infty\right]\right\} < 1$$

holds if and only if $\delta < M(A)^{-1}$. Namely there exists a network resource allocation $\{\delta_i, \delta_2, \ldots, \delta_m\}$ satisfying $\prod_{i=1}^m \delta_i = \delta$ such that the networked control system is stabilizable by state feedback if and only if $\delta < M(A)^{-1}$.

Proof: We will first show that if there exist a stabilizing F and a nonsingular diagonal D such that

$$\|D^{-1}T(z)DD_\delta\|_\infty < 1, \tag{6}$$

then

$$\delta = \prod_{i=1}^m \delta_i < M(A)^{-1}. \tag{7}$$

In the second part we will show that if $\delta < M(A)^{-1}$, then we can construct a stabilizing F, a real diagonal nonsingular matrix D, and a factorization of the form $\delta = \prod_{i=1}^m \delta_i$ such that the inequality (6) holds.

To simplify the proof, we assume that A has no eigenvalues on the unit circle. This assumption can be removed following the same argument as in [6], [8], [9]. Without loss of generality, realization matrices (A, B, F) are assumed to have the following decomposition:

$$A = \begin{bmatrix} A_{\rm u} & 0 \\ 0 & A_{\rm s} \end{bmatrix}, \;\; B = \begin{bmatrix} B_{\rm u} \\ B_{\rm s} \end{bmatrix}, \;\; F = \begin{bmatrix} F_{\rm u} & F_{\rm s} \end{bmatrix} \quad (8)$$

with compatible partition, where $A_{\rm s}$ is stable and $A_{\rm u}$ is anti-stable. As in the single input case, $F_{\rm s} = 0$ can be taken in minimizing the complementary sensitivity [9], and thus

$$T(z) = F_{\rm u}(zI - A_{\rm u} - B_{\rm u}F_{\rm u})^{-1}B_{\rm u} \quad (9)$$

can also be assumed in the proof. Consequently, we will simply assume that A is anti-stable.

To show that (6) implies (7) under the constraint that $A + BF$ is stable, we rewrite

$$D^{-1}T(z)D = F_{\rm D}(zI - A - B_{\rm D}F_{\rm D})^{-1}B_{\rm D} \quad (10)$$

with $F_{\rm D} = D^{-1}F$ and $B_{\rm D} = BD$. Then Lemma 3.1 can be applied to conclude that (6) is equivalent to the existence of $X \geq 0$ such that

$$X = A'X\left[I + B_{\rm D}(I - D_\delta^2)B_{\rm D}'X\right]^{-1}A \quad (11)$$

and

$$I > D_\delta B_{\rm D}'XB_{\rm D}D_\delta. \quad (12)$$

By the stabilizability of $[A|B]$, the matrix X, if exists, has a closed form expression:

$$X = \left(\sum_{k=1}^{\infty} A^{-k}B_{\rm D}(I - D_\delta^2)B_{\rm D}'A'^{-k}\right)^{-1}. \quad (13)$$

The above verifies that the solution $X > 0$ satisfies

$$X^{-1} = A_{\rm u}^{-1}X^{-1}A'^{-1} + A^{-1}B_{\rm D}(I - D_\delta^2)B_{\rm D}'A'^{-1} \quad (14)$$

Now pre- and post-multiplying both sides of inequality (12) by $\sqrt{D_\delta^{-2} - I}$, we obtain

$$D_\delta^{-2} - I > \sqrt{I - D_\delta^2}B_{\rm D}'XB_{\rm D}\sqrt{I - D_\delta^2}$$

Therefore if the condition (6) holds, the ARE in (11) has a solution $X > 0$ satisfying the above inequalities. Together with properties of determinant implies

$$\begin{aligned} \det(D_\delta^{-2}) &= \prod_{k=1}^{m} \delta_k^{-2} \\ &> \det\left(I + \sqrt{I - D_\delta^2}B_{\rm D}'XB_{\rm D}\sqrt{I - D_\delta^2}\right) \\ &= \det\left(I + B_{\rm D}(I - D_\delta^2)B_{\rm D}'X\right) \\ &= \det(AX^{-1}A'X) \\ &= \det(A)\det(A') = M(A)^2 \end{aligned}$$

which verifies inequality (7), completing one direction of the proof.

To show the converse part, we will seek a positive diagonal matrix D, stabilizing state feedback gain F, and a factorization $d = \prod_{i=1}^m \delta_i$ such that (6) holds. Without loss of generality, $[A|B]$ is assumed to have the following Wonham decomposition [18]:

$$A = \begin{bmatrix} A_1 & * & \cdots & * \\ 0 & A_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & * \\ 0 & \cdots & 0 & A_m \end{bmatrix}, \;\; B = \begin{bmatrix} b_1 & * & \cdots & * \\ 0 & b_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & * \\ 0 & \cdots & 0 & b_m \end{bmatrix}$$

with each pair $[A_i|b_i]$ stabilizable by the stabilizability of $[A|B]$. We now set

$$D = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & \epsilon & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \epsilon^{m-1} \end{bmatrix}$$

with ϵ a small real number. Also define

$$S = \begin{bmatrix} I_{n_1} & 0 & \cdots & 0 \\ 0 & \epsilon I_{n_2} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \epsilon^{m-1}I_{n_m} \end{bmatrix}.$$

Then

$$\begin{aligned} &D^{-1}T(z)DD_\delta \\ &= F_{\rm D}(zI - A - B_{\rm D}F_{\rm D})^{-1}B_{\rm D}D_\delta \\ &= F_{\rm D}S(zI - S^{-1}AS - S^{-1}B_{\rm D}F_{\rm D}S)^{-1}S^{-1}B_{\rm D}D_\delta \end{aligned}$$

where

$$S^{-1}AS = \begin{bmatrix} A_1 & o(\epsilon) & \cdots & o(\epsilon) \\ 0 & A_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & o(\epsilon) \\ 0 & \cdots & 0 & A_m \end{bmatrix},$$

$$S^{-1}B_{\rm D} = \begin{bmatrix} b_1 & o(\epsilon) & \cdots & o(\epsilon) \\ 0 & b_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & o(\epsilon) \\ 0 & \cdots & 0 & b_m \end{bmatrix}$$

and $\frac{o(\epsilon)}{\epsilon}$ approaches to a finite constant as $\epsilon \to 0$. Since $\delta < M(A)^{-1} = \prod_{i=1}^m M(A_i)^{-1}$, it is always possible to choose $\delta_i < M(A_i)^{-1}$ such that $\delta = \prod_{i=1}^m \delta_i$. We now set $F = F_{\rm D}S = \mathrm{diag}(f_1, f_2, \cdots, f_m)$ such that $A_i + b_if_i$ is stable for all $1 \leq i \leq m$ and $\|f_i(zI - A_i - b_if_i)^{-1}b_i\|_\infty < \delta_i^{-1}$. Such an f_i exists by Theorem 2.1 and the fact that $\delta_i^{-1} > M(A_i)$. It can now be verified that

$$\begin{aligned} &D^{-1}T(z)DD_\delta \\ &= \mathrm{diag}\left(T_1(z)\delta_1, T_2(z)\delta_2, \ldots, T_m(z)\delta_m\right) + o(\epsilon; z) \end{aligned}$$

where $T_i(z) = f_i(zI - A_i - b_if_i)^{-1}b_i$, and $o(\epsilon; z) \to 0$ as $\epsilon \to 0$ for each $|z| \geq 1$. Since $\|T_i(s)\delta_i\|_\infty < 1$, it follows that $\|D^{-1}T(z)DD_\delta\|_\infty < 1$ for sufficiently small ϵ. □

The sufficiency proof is constructive. Let the set of eigenvalues of A that are controllable by the i-th input but not by any of the first, second, ... and $(i-1)$-st inputs be Λ_i. Let $M_i(A)$ be the absolute product of the numbers in Λ_i outside of the unit circle. Then the stabilizability of $[A|B]$ implies that $\cup_{i=1}^m \Lambda_i$ contains all unstable eigenvalues and $M(A) = \prod_{i=1}^m M_i(A)$. For a $\delta < M(A)^{-1}$, a feasible allocation of $\delta_, \delta_2, \ldots, \delta_m$ so that $\delta = \prod_{i=1}^m \delta_i$ given in the proof is to make $\delta_i < M_i(A)^{-1}$. Clearly, such an allocation always exists. In other words, the allocation of d_i was done as follows: choose d_1 so that the first input can be used to stabilize all unstable modes controllable from the first input, then choose d_2 so that the second input can be used to stabilize the additional unstable modes controllable from the second input excluding the ones that are already stabilized by the first input, ... , finally d_m is chosen so to stabilize the remaining unstable modes that are not stabilized by the other inputs. This is exactly the sequential design idea used in the first multi-input pole placement solution in [18]. The choice of D also reinforces this idea. For small ϵ, D has the interpretation that the first input is utilized in full to do the stabilization as much as possible, then the second input is utilized in full to do the leftover job of the first input as much as possible, then continue with the third and fourth inputs, ..., finally the m-th input is utilized in full to do the remaining job that the other inputs cannot do. With such an allocation of $\delta_1, \delta_2, \ldots, \delta_m$ and such a choice of weighting matrix D, a feasible feedback gain F is designed as a state feedback $\mathcal{H}_\infty$ satisfying

$$\|D^{-1}F(z)DD_\delta\|_\infty < 1.$$

Notice that the re-ordering of inputs does not affect the above scheme, but the sequential design results in a different D and D_δ. Hence, the resource allocation and feedback gain design is not unique. However, no matter how the re-ordering is applied, there is a minimum resource which has to be allocated to the i-th channel. This is given by the unstable modes only controllable by the input i. This is the minimum stabilization work that input i has to accomplish no matter how the design is carried out.

The control problem with resource allocation can be considered as a case of combined plant/controller design. It reiterates that the controller designer should not just passively take what the system designer gives. A controller designer should participate in the system design so that the controller design becomes easy. The issue of system design for easy controller design has been studied before. Our result here strengthens the importance of this issue.

IV. Conclusion

The problem of stabilization of multi-input LTI systems via transmission channels is considered. The transmission channels are modelled as sector uncertainties, which is partially motivated by the problem of stabilization with logarithmically quantized inputs studied in [8], [9]. The single-input results in [8], [9] on finding the smallest amount of information needed in the input channel so that the stabilization is possible are extended to multi-input systems by solving a modified μ-synthesis problem analytically. The solution is given in terms of the Mahler measure of the plant.

Only state feedback control is considered in this paper. This serves as the starting point and it is also of fundamental importance. The output feedback networked control is much more challenging than the one investigated in this paper, and is currently under our study.

Acknowledgment

The authors would like to thank Professor Weizhou Su of South China University of Technology for interesting discussions.

This research is partially supported by US Air Force and the Research Grants Council of Hong Kong.

References

[1] R. Bowen, "Entropy for group endomorphisms and homogeneous space", *Trans. of the American Mathematical Society*, vol 153, pp. 401-414, 1971.

[2] Special Issue on Networked Control Systems, *IEEE Trans. Automat. Contr.*, vol. 49, Sept. 2004.

[3] Special Issue on Technology of Networked Control Systems, *Proc. of IEEE*, vol. 95. Jan. 2007.

[4] J. Baillieul, "Feedback coding for information based control: Operating near the data rate limit," in *Proc. 41st IEEE Conf. Decision Contr.*, Las Vegas, NV, 2002.

[5] T. Başar and P. Bernhard, *$\mathcal{H}_\infty$ Optimal Control and Related Minimax Design Problems: A Dynamic Gain Approach*, Systems and Control: Foundations and Applications, Birkhäuser, Boston, MA, 1991.

[6] J. H. Braslavsky, R. H. Middleton, and J. S. Freudenberg, "Feedback stabilization over signal-to-noise ratio constrained channels," *IEEE Trans. Automat. Contr.*, vol. 52, pp. 1391-1403, Aug. 2007.

[7] J. Doyle, "Analysis of feedback systems with structured uncertainty," *IEE Proceedings*, Part D, vol. 129, pp. 242-250, Nov. 1982.

[8] N. Elia and S. K. Mitter, "Stabilization of linear systems with limited information," *IEEE Trans. Automat. Contr.*, vol. 46, pp. 1384-1400, Sept. 2001.

[9] M. Fu and L. Xie, "The sector bound approach to quantized feedback control," *IEEE Trans. Automat. Contr.*, vol. 50, pp. 1698-1711, Nov. 2005.

[10] G. C. Goodwin, D. E. Quevedo and E. I. Silva, "An introduction to networked control systems", *Proceedings of Asian Control Conference*, pp. 50-74, 2006.

[11] M. Green and D. J. L. Limebeer, *Linear Robust Control*, Prentice-Hall, 1995.

[12] P. P. Khargonekar, I. R. Petersen, and K. Zhou, "Robust stabilization of uncertain linear systems: Quadratic stabilizability and $\mathcal{H}_\infty$ control theory", *IEEE Trans. Automat. Contr.*, vol. 35, pp. 356-361, Mar. 1990.

[13] K. Mahler, "An application of Jensen's formula to polynomials", *Mathematika*, vol. 7, pp. 98-100, 1960.

[14] G. N. Nair, F. Fagnani, S. Sampieri, and R. J. Evans, "Feedback control under data rate constraints: an overview", *Proc. of IEEE*, vol. 95, pp. 108-137, 2007.

[15] G. N. Nair and R. J. Evans, "Exponential stabilizability of finite-dimensional linear systems with limited data rates," *Automatica*, vol. 39, pp. 585-593, 2003.

[16] J. S. Shamma, "Robust stability with time-varying structured uncertainty", *IEEE Trans. Automat. Contr.*, vol. 39, pp. 714-724, 1994.

[17] B. Sklar, *Digital Communications – Fundamentals and Applications*, Prentice-Hall, Upper Saddle River, New Jersey, 2000.

[18] W. M. Wonham, "On pole assignment in multi-input controllable linear systems", *IEEE Trans. Automat. Contr.*, vol. AC-12, pp. 660-665, 1967.