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Time and Event-based Sensor Scheduling for Networks with Limited Communication Resources

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Abstract: In this paper, we consider state estimation over a network subject to limited sensor communications. A sensor needs to decide when to send its local state estimate to a remote estimator in order to minimize the average estimation error at the estimator subject to that the total communication time is no more than a pre-specified value. We propose a novel sensor schedule that combines conventional time and event-based methods and demonstrate that the estimator performance is improved compared with the optimal time-based schedule and the computation complexity is reduced compared with the optimal event-based schedule. Thus the proposed schedule provides a tradeoff between the two classic approaches.

1. INTRODUCTION

The last decade has seen a growing interest in the area of networked control systems (Hespanha et al. [2007]), which, thanks to the recent advances in network infrastructure, communication architecture and computer technology, have a broad range of applications including transportation, health care systems, agriculture, smart home and smart grid, etc. New issues, however, arise when the control loop is closed over a network. For example, network induced delays and data packet drops may severely degrade system performance and may even cause instability. Shared resources often imply no dedicated communication paths between key components of the closed-loop control system. Consequently sensor measurement data may not be sent to the controller and control data may not arrive at actuator at each time. If the resource is severely limited, system performance will again be degraded and cannot be guaranteed.

This paper focuses on the analysis and design of a networked state estimator subject to limited communication resources. Specifically, we consider the scenario when a sensor can only communicate with a remote state estimator m times within a time-horizon $T \gg m$. Before we present the main result of this paper, we briefly review some related works in literature.

Savage and Scala [2009] studied a special class of scalar systems and proposed a solution to when should the sensor send its measurement data so that the terminal error covariance at the estimator is minimized. Mo et al. [2009] considered when should the sensor send its measurement data so that the average error covariance at the estimator is minimized. By using convex relaxation techniques, a suboptimal sensor schedule is given. A stochastic sensor scheduling scheme was proposed in (Gupta et al. [2006]) and these authors provided the optimal probability distribution over the sensors to be selected so that the expected error covariance is minimized. Sandberg et al. [2008] considered estimation over a heterogeneous sensor network. Two types of sensors were investigated: the first type has low-quality measurement but small processing delay, while the second type has high-quality measurement but large processing delay. Using a time-periodic Kalman filter, they showed how to find an optimal schedule of the sensor communication.

The aforementioned work can be classified as finding the optimal time-based sensor schedules. In other words, the sensor schedules are computed before the system is run and only depend on the running time. There is yet another important class of schedules known as event-based schedules. That is the time for the sensor to communicate with the estimator depends on whether a pre-defined event happens or not, e.g., the predicted estimation error grows beyond a certain threshold. By properly designing the event-triggering mechanism, the performance of the estimator is often superior compared to using time-based schedules.

Åström and Bernhardsson [2002] considered a simple first order stochastic system. They showed that for the same average sampling rate, the event-based approach leads to smaller state variance. A constant factor approximation algorithm for event-based sampling was proposed by Cogill et al. [2007]. The resulting sensor communication schedule showed a tradeoff between communication rate and estimation error. Real-time scheduling of stabilizing control tasks was revisited in (Tabuada [2007]) and an event-based approach was proposed which leads to guaranteed performance while relaxing the traditional periodic execution requirements. Rabi et al. [2008] presented a novel event-triggered sensing and actuation strategy for a class of networked control systems where, within a given time interval, the control signal can change values no more than a prescribed number of times. In simple situations, the authors were able to analytically derive the optimal time-varying event detector. Ramesh et al. [2009] proposed a novel architecture for control over wireless networks by integrating the design of the media access control protocol. Different random access methods were classified and an adaptive random access one using an event-triggering mechanism for determining channel access was identified. In (Li et al. [2010]) an event-triggered approach was used to trigger the data transmission from a sensor to a remote observer. The event-triggering rule was designed to compute the minimum mean squared error (MMSE) of the state estimate at the remote observer subject to a constraint on how frequently the information can be transmitted. Computing the optimal event-trigger rule was shown to be computationally intractable when the state dimension exceeds two or when the time-horizon is large, in which cases, suboptimal rules were computed instead. A related problem was considered by Imer and Basar Imer and Basar [2010] where one observer agent needs to communicate its data with an estimator agent to minimize the estimation error at the estimator subject to the constraint that a limited number of communications is allowed in between the observer and the estimator.

The main contribution of this paper is the introduction of a novel sensor data scheduling architecture that is both time and event-based. We first construct the optimal time-based schedule under some mild assumptions. On top of this optimal time-based schedule, an event-triggering mechanism is introduced. Although this new schedule seems to be even more complicated than the time-based or event-based approach alone, by optimizing this event-triggering mechanism, it is shown to have better performance than time-based schedules and are computationally cheaper than event-based ones. Thus the hybrid schedule provides a tradeoff between these two classes of schedules.

The rest of the paper is organized as follows. Mathematical models of the system are given in Section 2. The optimal time-based schedule is given in Section 3. Based on this optimal time-based schedule, a hybrid schedule is constructed in Section 4 and is shown to have better performance. Some concluding remarks are given in the end.

Notations: $\mathbb{Z}$ is the set of non-negative integers. $\mathbb{N}$ is the set of natural numbers. $k \in \mathbb{Z}$ is the time index. $\mathbb{R}^n$ is the n dimensional Euclidean space. $\mathbb{S}_+^n$ is the set of n by n
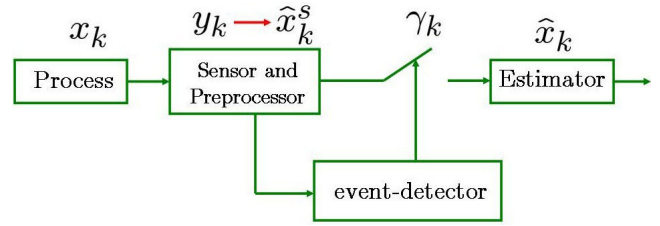

Fig. 1. Proposed hybrid scheduling architecture for remote estimation.

positive semi-definite matrices. When $X \in \mathbb{S}_+^n$, it is written as $X \geq 0$. $X \geq Y$ if $X - Y \in \mathbb{S}_+^n$. $\mathbb{E}[\cdot]$ is the expectation of a random variable and $\mathbb{E}[\cdot|\cdot]$ is the conditional expectation. $\Pr(\cdot)$ is the probability of a random event. $\mathrm{Tr}(\cdot)$ is the trace of a matrix. For functions $f, f_1, f_2 : \mathbb{S}_+^n \to \mathbb{S}_+^n$, $f_1 \circ f_2$ is defined as $f_1 \circ f_2(X) \triangleq f_1\big(f_2(X)\big)$ and f^t is defined as $f^t(X) \triangleq \underbrace{f \circ f \circ \cdots \circ f}_{t \text{ times}}(X)$.

2. PROBLEM SETUP

2.1 System Models

Consider the following discrete linear time-invariant process (Fig. 1)

$$x_{k+1} = Ax_k + w_k, \qquad (1)$$
$$y_k = Cx_k + v_k, \qquad (2)$$

where $x_k \in \mathbb{R}^{n_x}$ is the process state vector, $y_k \in \mathbb{R}^{n_y}$ is the observation vector, $w_k \in \mathbb{R}^{n_x}$ and $v_k \in \mathbb{R}^{n_y}$ are zero-mean Gaussian random vectors with $\mathbb{E}[w_k w_j{}'] = \delta_{kj}Q \geq 0$, $\mathbb{E}[v_k v_j{}'] = \delta_{kj}R > 0$, $\mathbb{E}[w_k v_j{}'] = 0 \ \forall j, k$. The initial state x_0 is a zero-mean Gaussian random vector that is uncorrelated with w_k and v_k and has covariance $\Pi_0 \geq 0$. The pair (A, C) is assumed to be observable and $(A, \sqrt{Q})$ is controllable.

Assume the sensor runs a Kalman filter to compute $\hat{x}_k^s$, the local MMSE estimate of x_k in (1). Let e_k^s and P_k^s be the estimation error and error covariance matrix, i.e.,

$$e_k^s \triangleq x_k - \hat{x}_k^s, \qquad (3)$$
$$P_k^s \triangleq \mathbb{E}[(e_k^s)(e_k^s)'|y_0, \ldots, y_k], \qquad (4)$$

which are computed through the following equations:

$$\hat{x}_{k|k-1}^s = A\hat{x}_{k-1}^s, \qquad (5)$$
$$P_{k|k-1}^s = AP_{k-1}^s A' + Q, \qquad (6)$$
$$K_k = P_{k|k-1}^s C'[CP_{k|k-1}^s C' + R]^{-1}, \qquad (7)$$
$$\hat{x}_k^s = A\hat{x}_{k-1}^s + K_k(y_k - CA\hat{x}_{k-1}^s), \qquad (8)$$
$$P_k^s = (I - K_k C)P_{k|k-1}^s, \qquad (9)$$

where the recursion starts from $\hat{x}_0^s = 0$ and $P_0^s = \Pi_0$.

After $\hat{x}_k^s$ is obtained, the sensor decides whether to send it to the remote estimator. Let γ_k be the decision variable at time k, i.e., if $\gamma_k = 1$, then $\hat{x}_k^s$ is sent, and if $\gamma_k = 0$, $\hat{x}_k^s$ is not sent. Define a schedule θ as

$$\theta = \{\gamma_1, \ldots, \gamma_T\} \in \{0, 1\}^T.$$

Under a given θ, the remote estimator calculates $\hat{x}_k$ and P_k: its estimate of x_k and the associated error covariance. The procedure of calculating $\hat{x}_k$ and P_k depends on θ and will be introduced in subsequent sections.

Let $T \in \mathbb{N}$ be the time-horizon, and define $J(\theta)$ as the trace of the average expected estimation error covariance, i.e.,

$$J(\theta) \triangleq \frac{1}{T} \sum_{k=1}^{T} \mathrm{Tr}\left(\mathbb{E}\left[P_k(\theta)\right]\right). \tag{10}$$

We are interested in finding a schedule $\theta \in \{0,1\}^T$ that solves the following problem.

Problem 2.1.

$$\min_{\theta} J(\theta)$$

$$\text{s.t.} \sum_{k=1}^{T} \gamma_k(\theta) = m$$

where $m \ll T$ denotes the maximum number of times that the sensor can communicate with the remote estimator.

The constraint is motivated by, for example, limited communication energy at the sensor or limited bandwidth of the network. We consider T sufficiently large so that the time needed by the Kalman filter at the sensor to enter steady state is negligible when compared with T. Notice that this is not such a restrictive assumption as P_k^s typically converges to its steady-state value, $\overline{P}$, exponentially fast. Under this assumption, we may assume $\Pi_0 = \overline{P}$. As a result, one easily obtains

$$P_k^s = \overline{P}, \ K_k = K = \overline{P}C'R^{-1}, \ \forall\, k \geq 1. \tag{11}$$

2.2 Kalman Filtering Preliminaries

Before we state the main results of the paper, we provide a brief summary of some properties of the Kalman filter. Define the following functions on $\mathbb{S}_+^n$. First define $h : \mathbb{S}_+^n \to \mathbb{S}_+^n$ as

$$h(X) \triangleq AXA' + Q. \tag{12}$$

Applying h to the previous error covariance matrix P_{k-1}^s corresponds to the time update of the Kalman filter. Similarly, define the function $\tilde{g} : \mathbb{S}_+^n \to \mathbb{S}_+^n$ as

$$\tilde{g}(X) \triangleq X - XC'[CXC' + R]^{-1}CX. \tag{13}$$

Applying $\tilde{g}$ to $h(P_{k-1}^s)$ corresponds to the measurement update of the Kalman filter. It is straightforward to verify that the following (e.g., Lemma A.1 in Shi et al. [2010]).

$$h(X) \leq h(Y), \ \tilde{g}(X) \leq \tilde{g}(Y), \tilde{g}(X) \leq X, \ \forall\, 0 \leq X \leq Y.$$

Some properties of e_k^s defined in (3) are summarized in the next lemma.

Lemma 2.2. The following statements on e_k^s hold:

(1) e_k^s is independent of $\hat{x}_k^s$, hence $\mathbb{E}[(e_k^s)(\hat{x}_k^s)'] = 0$.
(2) e_k^s is independent of w_{k_1} and v_{k_2} for any $k_1, k_2 \in \mathbb{N}$ and $k_1 \geq k, k_2 \geq k+1$.
(3) e_k^s is independent of $\hat{x}_k^s - A^d \hat{x}_{k-d}^s$ for any $d \in \mathbb{N}$.
(4) e_k^s is zero-mean Gaussian.

Proof: (1) Direct result from the orthogonality principle (Kailath et al. [2000]). (2) Write e_k^s as

$$e_k^s = (A - KCA)e_{k-1}^s + (I - KC)w_{k-1} - Kv_k. \tag{14}$$

Thus e_k^s is a linear function of $x_0, w_0, \ldots, w_{k-1}$, and $v_1, \ldots, v_k$. Since x_0, w_k's and v_k's are mutually independent, the statement holds. (3) From (14), we see that e_k^s is also a linear function of $e_{k-d}^s, w_{k-d}, \ldots, w_{k-1}$, and $v_{k-d+1}, \ldots, v_k$. From (8), $\hat{x}_{k-d}^s$ only depends on x_0, $w_0, \ldots, w_{k-d-1}$, and $v_1, \ldots, v_{k-d}$, thus $\hat{x}_{k-d}^s$ is independent of $w_{k-d}, \ldots, w_{k-1}$ and $v_{k-d+1}, \ldots, v_k$. From the first statement, $\hat{x}_{k-d}^s$ is independent of e_{k-d}^s. Therefore we conclude that $\hat{x}_{k-d}^s$ is independent of e_k^s. Together with the first statement, we arrive at the fact that e_k^s is independent of $\hat{x}_k^s - A^d \hat{x}_{k-d}^s$. (4) Since x_0, w_k's and v_k's are all zero-mean Gaussian, from (14), e_k^s is also zero-mean Gaussian. ∎

With some manipulation, P_k^s can be shown to satisfy $P_k^s = \tilde{g} \circ h(P_{k-1}^s)$. Furthermore, the steady-state error covariance, $\overline{P}$, is the unique positive semi-definite solution of $\tilde{g} \circ h(X) = X$ (see Anderson and Moore [1979]).

Lemma 2.3. For $0 \leq t_1 \leq t_2$, the following inequality holds:

$$h^{t_1}(\overline{P}) \leq h^{t_2}(\overline{P}). \tag{15}$$

In addition, if $t_1 < t_2$, then

$$\mathrm{Tr}\left(h^{t_1}(\overline{P})\right) < \mathrm{Tr}\left(h^{t_2}(\overline{P})\right). \tag{16}$$

Proof: First notice that $\overline{P} = \tilde{g} \circ h(\overline{P}) \leq h(\overline{P})$. Therefore by applying h on both sides of the inequality we get $\overline{P} \leq h(\overline{P}) \leq h^2(\overline{P})$. Repeating the same procedure, we obtain

$$\overline{P} \leq h(\overline{P}) \leq \cdots \leq h^{t-1}(\overline{P}) \leq h^t(\overline{P}), \ \forall\, t \geq 0.$$

Next assume $\overline{P} = h(\overline{P})$. With some manipulation, we arrive at $Q = 0$, which contradicts with the assumption that $(A, \sqrt{Q})$ is controllable. Thus $\overline{P} \neq h(\overline{P})$. Consequently if $t_1 < t_2$, then $h^{t_1}(\overline{P}) \neq h^{t_2}(\overline{P})$. Therefore (16) holds. ∎

3. OPTIMAL TIME-BASED SENSOR SCHEDULE

In this section, we introduce the optimal time-based sensor schedule. At the estimator side, it is straightforward to show that the optimal state estimate and error covariance evolve as

$$(\hat{x}_k, P_k) = \begin{cases} (A\hat{x}_{k-1}, h(P_{k-1})), & \text{if } \gamma_k = 0, \\ (\hat{x}_k^s, P_k^s), & \text{if } \gamma_k = 1. \end{cases}$$

From (11), P_k is given by

$$P_k = \begin{cases} h(P_{k-1}), & \text{if } \gamma_k = 0, \\ \overline{P}, & \text{if } \gamma_k = 1. \end{cases}$$

For simplicity, we consider $m = 2t - 1$ and $T = 4qt - 1$ for $t, q \in \mathbb{N}$. Other forms of T and m can be dealt similarly. The next proposition states the optimal time-based schedule.

Proposition 3.1. The optimal time-based schedule $\theta_t^* \in \{0,1\}^T$ that minimizes $J(\theta_t)$ in (10) is given by:

$$\gamma_{2lq} = 1 \ \forall\, l = 1, \ldots, 2t - 1, \ \text{and} \ \gamma_k = 0 \ \text{otherwise}.$$

The corresponding minimum $J(\theta_t)$ is given by

$$J(\theta_t^*) = \frac{2t}{T} \sum_{i=0}^{2q-1} \mathrm{Tr}\left(h^i(\overline{P})\right) - \frac{1}{T}\mathrm{Tr}\left(\overline{P}\right). \tag{17}$$

Proof: It is straightforward to verify that $J(\theta_t^*)$ for the θ_t^* defined in the proposition is indeed given by (17). Next we

prove $J(\theta_t^*) \leq J(\theta_t)$ for any schedule θ_t. Notice that there are exactly $m + 1 = 2t$ instances where $P_k = \overline{P}$ including P_0. Introduce $k_j \in \mathbb{N}, j = 1, \ldots, 2t$ for θ_t such that $TJ(\theta_t)$ can be written as

$$TJ(\theta_t) = \mathrm{Tr}\left[\sum_{j=1}^{2t}\sum_{i=0}^{k_j} h^i(\overline{P}) - \overline{P}\right].$$

Clearly k_j's satisfy

$$\sum_{j=1}^{2t} k_j = 2t(2q - 1) \qquad (18)$$

since $T = 4qt - 1$. k_j's can be divided into three groups $\mathcal{G}_i, i = 1, 2, 3$ as follows:

(1) $k_j \in \mathcal{G}_1$ if $k_j < 2q - 1$;
(2) $k_j \in \mathcal{G}_2$ if $k_j = 2q - 1$;
(3) $k_j \in \mathcal{G}_3$ if $k_j > 2q - 1$.

Then from (18),

$$0 = \sum_{j=1}^{2t} k_j - 2t(2q - 1)$$
$$= \sum_{k_j \in \mathcal{G}_1} (k_j - 2q + 1) + \sum_{k_j \in \mathcal{G}_3} (k_j - 2q + 1).$$

Therefore

$$\sum_{k_j \in \mathcal{G}_1} (2q - 1 - k_j) = \sum_{k_j \in \mathcal{G}_3} (k_j - 2q + 1). \qquad (19)$$

Now

$$T\left(J(\theta_t) - J(\theta_t^*)\right)$$
$$= \mathrm{Tr}\left[\sum_{j=1}^{2t}\sum_{i=0}^{k_j} h^i(\overline{P}) - 2t\sum_{i=0}^{2q-1} h^i(\overline{P})\right]$$
$$= \mathrm{Tr}\left[\sum_{k_j \in \mathcal{G}_3}\sum_{i=2q}^{k_j} h^i(\overline{P}) - \sum_{k_j \in \mathcal{G}_1}\sum_{i=k_j+1}^{2q-1} h^i(\overline{P})\right]$$
$$\geq \mathrm{Tr}\left[\sum_{k_j \in \mathcal{G}_3}\sum_{i=2q}^{k_j} h^{2q}(\overline{P}) - \sum_{k_j \in \mathcal{G}_1}\sum_{i=k_j+1}^{2q-1} h^{2q}(\overline{P})\right]$$
$$= \left[\sum_{k_j \in \mathcal{G}_3} (k_j - 2q + 1) - \sum_{k_j \in \mathcal{G}_1} (2q - 1 - k_j)\right]\mathrm{Tr}\left(h^{2q}(\overline{P})\right)$$
$$= 0,$$

where the inequality is from Lemma 2.3 and the last equality is from (19). Thus the proof is complete. ∎

Remark 3.2. The optimal time-based schedule is a periodic schedule with period $2q$ and the m communication times between the sensor and the remote estimator are separated *as uniform as possible*. This principle holds for general m and T.

4. A HYBRID SENSOR SCHEDULE

In this section, we construct a hybrid sensor schedule, which includes an event-triggering mechanism on top of the optimal time-based sensor schedule θ_t^* defined in Proposition 3.1. We first introduce this hybrid schedule and then compare its performance with that of θ_t^*.

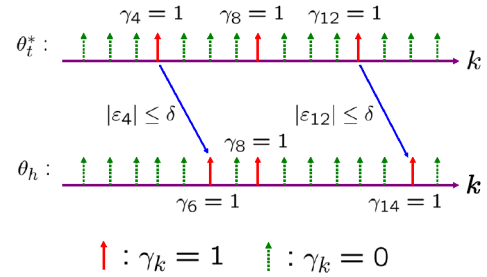
Fig. 2. Realization of θ_t^* and θ_h

4.1 A Hybrid Sensor Schedule

First note that the sensor is able to calculate $\hat{x}_k$ as it has access to all γ_k's. Define ε_k as

$$\varepsilon_k \triangleq \hat{x}_k^s - A\hat{x}_{k-1}, \qquad (20)$$

where $A\hat{x}_{k-1}$ is the predicted state estimate at the estimator based on the previous optimal state estimate $\hat{x}_{k-1}$. Thus if $\hat{x}_k^s$ is not sent at time k, ε_k will indicate how close is the state estimate at the estimator from the optimal state estimate at the sensor. If the size of ε_k is small, then we are certain that even without receiving $\hat{x}_k^s$, the state estimate at the estimator is very close to $\hat{x}_k^s$. This observation inspires us to consider the following hybrid sensor schedule θ_h: *for a given threshold $\delta \geq 0$, θ_h is identical to θ_t^* except at $k = 2lq$ when l is odd, in which instances, if $|\varepsilon_{2lq}| \leq \delta$, set $\gamma_{2lq} = 0$ and set $\gamma_{(2l+1)q} = 1$.*

The remote estimator remains

$$\hat{x}_k = \begin{cases} A\hat{x}_{k-1}, & \text{if } \gamma_k = 0, \\ \hat{x}_k^s, & \text{if } \gamma_k = 1. \end{cases}$$

However, as we shall see shortly in Lemma 4.3, the associated error covariance P_k will take a different form than that under θ_t^*.

Due to the random process and measurement noise, the instances for the sensor to send $\hat{x}_k^s$ to the remote estimator under θ_h are random, while they are fixed under θ_t^*. Fig. 2 shows a particular realization of the sensor communication times of θ_h for $t = 2$, $q = 2$, $m = 3$ and $T = 15$. For comparison purpose, the sensor communication times for θ_t^* are also shown.

4.2 Comparison of θ_h with θ_t^*

In this section we compare the performance of θ_h with that of θ_t^*. The following few lemmas are useful to derive the main result of this section.

Lemma 4.1. ε_{2lq} is zero-mean Gaussian and its covariance is independent of l.
Proof: First note that

$$\varepsilon_{2lq} = \hat{x}_{2lq}^s - A\hat{x}_{2lq-1} = \hat{x}_{2lq}^s - A^{2q}\hat{x}_{2(l-1)q}$$
$$= \hat{x}_{2lq}^s - A^{2q}\hat{x}_{2(l-1)q}$$
$$= Me_{2(l-1)q}^s + \sum_{i=1}^{2q} \Phi_i w_{2lq-i} + \sum_{i=0}^{2q-1} \Psi_i v_{2lq-i},$$

where simple calculation shows that

$$M = A^{2q} - (A - KCA)^{2q},$$

$$\Phi_1 = KC,$$

$$\Phi_{i+1} = (A - KCA)\Phi_i + KCA^i, \ i = 1, 2, \ldots, 2q - 1,$$

$$\Psi_i = (A - KCA)^i K.$$

From Lemma 2.2, $e_{2(l-1)q}^s$, $w_{2(l-1)q}$, ..., w_{2lq-1}, and $v_{2(l-1)q+1}, \ldots, v_{2lq}$ are all mutually independent zero-mean and Gaussian random variables, hence we conclude that ε_{2lq} for an odd number l is also zero-mean and Gaussian and has covariance

$$\mathbb{E}[\varepsilon_{2lq}\varepsilon_{2lq}'] = M\overline{P}M' + \sum_{i=1}^{2q} \Phi_i Q \Phi_i' + \sum_{i=0}^{2q-1} \Psi_i R \Psi_i'$$

which does not depend on l. ∎

Since the distribution of ε_{2lq} is independent of l, Δ_δ defined by

$$\Delta_\delta = \mathbb{E}[\varepsilon_{2lq}\varepsilon_{2lq}' | |\varepsilon_{2lq}| \leq \delta] \tag{21}$$

is also independent of l.

Lemma 4.2. $\mathbb{E}[e_{2lq}^s \varepsilon_{2lq}'] = 0$.

Proof: Since $\varepsilon_{2lq} = \hat{x}_{2lq}^s - A^{2q}\hat{x}_{2(l-1)q}^s$, from part (3) of Lemma 2.2, e_{2lq}^s is independent of ε_{2lq}. Therefore

$$\mathbb{E}[e_{2lq}^s \varepsilon_{2lq}'] = \mathbb{E}[e_{2lq}^s]\mathbb{E}[\varepsilon_{2lq}'] = 0.$$

∎

Lemma 4.3. The error covariance P_k under θ_{h} has the same form as that under θ_{t}^*, i.e.,

$$P_k = \begin{cases} h(P_{k-1}), & \text{if } \gamma_k = 0, \\ \overline{P}, & \text{if } \gamma_k = 1, \end{cases}$$

except at those time instances $k = 2lq$ when l is odd and $|\varepsilon_k| \leq \delta$, in which cases, $\gamma_{2lq} = 0$ and P_k is given by

$$P_k = \overline{P} + \Delta_\delta. \tag{22}$$

Proof: We only prove the exceptional case as other cases are straightforward to see. At the estimator, if no packet is received at time $k = 2lq$, then $\gamma_{2lq} = 0$, which implies $|\varepsilon_{2lq}| \leq \delta$. Since $\hat{x}_{2lq}^s = A\hat{x}_{2lq-1}$, we have $|\hat{x}_{2lq}^s - \hat{x}_{2lq}| \leq \delta$. Therefore

$$\begin{aligned}
P_{2lq} &= \mathbb{E}[(x_{2lq} - \hat{x}_{2lq})(\cdot)' | |\varepsilon_{2lq}| \leq \delta] \\
&= \mathbb{E}[(x_{2lq} - \hat{x}_{2lq}^s + \hat{x}_{2lq}^s - \hat{x}_{2lq})(\cdot)' | |\varepsilon_{2lq}| \leq \delta] \\
&= \mathbb{E}[(e_{2lq}^s + \varepsilon_{2lq})(e_{2lq}^s + \varepsilon_{2lq})' | |\varepsilon_{2lq}| \leq \delta] \\
&= \mathbb{E}[(e_{2lq}^s)(e_{2lq}^s)' | |\varepsilon_{2lq}| \leq \delta] + \mathbb{E}[\varepsilon_{2lq}\varepsilon_{2lq}' | |\varepsilon_{2lq}| \leq \delta] \\
&= \overline{P} + \Delta_\delta,
\end{aligned}$$

where the second last equality is from Lemma 4.2. ∎

Lemma 4.4. Δ_δ satisfies the following

(1) $\Delta_0 = 0$.
(2) Δ_δ is strictly increasing in δ.
(3) $\Delta_\delta \leq \delta^2 I$.
(4)
$$\lim_{\delta \to \infty} \Delta_\delta = h^{2q}(\overline{P}) - \overline{P}. \tag{23}$$

Proof: The first three statements are self-evident. When $\delta \to \infty$, the event $|\varepsilon_{2lq}| \leq \delta$ carries no new information, thus P_{2lq} is the same as that using the time-based schedule with $\gamma_{2lq} = 0$, i.e., $P_{2lq} = h^{2q}(\overline{P})$. On the other hand, we also have

$$\begin{aligned}
P_{2lq} &= \mathbb{E}[(e_{2lq}^s)(e_{2lq}^s)' | |\varepsilon_{2lq}| \leq \infty] + \mathbb{E}[\varepsilon_{2lq}\varepsilon_{2lq}' | |\varepsilon_{2lq}| \leq \infty] \\
&= \mathbb{E}[(e_{2lq}^s)(e_{2lq}^s)'] + \mathbb{E}[\varepsilon_{2lq}\varepsilon_{2lq}'] \\
&= \overline{P} + \Delta_\delta.
\end{aligned}$$

Therefore (23) holds. ∎

We now introduce the main result of this section. The theorem below provides a sufficient and necessary condition on δ such that θ_{h} outperforms θ_{t}^*.

Theorem 4.5. The following statements hold.

(1) $J(\theta_{\mathrm{h}}) \leq J(\theta_{\mathrm{t}}^*)$ iff $\delta \in [0, \delta_{\max}]$ where $\delta_{\max}$ is the unique solution to

$$\mathrm{Tr}\left[\sum_{i=q}^{2q-1} h^i(\overline{P})\right] = \mathrm{Tr}\left[\sum_{i=0}^{q-1} h^i(\overline{P} + \Delta_{\delta_{\max}})\right]. \tag{24}$$

A lower bound of $\delta_{\max}$ is given by

$$\delta_{\max} \geq \sqrt{\lambda_{\min}\left(h^q(\overline{P}) - \overline{P}\right)}. \tag{25}$$

(2) If $\delta_{\max} > 0$ is the solution of (24), then for any $\delta \in (0, \delta_{\max})$ and for any realization ϕ of θ_{h},

$$J(\phi) \leq J(\theta_{\mathrm{t}}^*). \tag{26}$$

Furthermore there exists a positive probability of ϕ such that (26) becomes strict, hence $J(\theta_{\mathrm{h}}) < J(\theta_{\mathrm{t}}^*)$.

Proof: (1) First note that for any odd number l,

$$\begin{aligned}
&\sum_{k=2(l-1)q}^{2(l+1)q-1} \mathbb{E}[P_k(\theta_{\mathrm{h}})] \\
&= \sum_{i=0}^{2q-1} h^i(\overline{P}) + \mathrm{Pr}\left(|\varepsilon_{2lq}| > \delta\right) \sum_{i=0}^{2q-1} h^i(\overline{P}) \\
&\quad + \mathrm{Pr}\left(|\varepsilon_{2lq}| \leq \delta\right) \left[\sum_{i=0}^{q-1} h^i(\overline{P} + \Delta_\delta) + \sum_{i=0}^{q-1} h^i(\overline{P})\right],
\end{aligned}$$

which in independent of l. Define D_δ as

$$D_\delta = \mathrm{Tr}\left[\sum_{i=q}^{2q-1} h^i(\overline{P}) - \sum_{i=0}^{q-1} h^i(\overline{P} + \Delta_\delta)\right].$$

With some manipulation, we obtain $J(\theta_{\mathrm{t}}^*) - J(\theta_{\mathrm{h}}) = \frac{1}{4q}\mathrm{Pr}\left(|\varepsilon_{2lq}| \leq \delta\right) D_\delta$. Since $\mathrm{Pr}\left(|\varepsilon_{2lq}| \leq \delta\right) \geq 0$, $J(\theta_{\mathrm{t}}^*) \geq J(\theta_{\mathrm{h}})$ iff $D_\delta \geq 0$. From Lemmas 2.3 and 4.4, D_δ is strictly decreasing in δ and

$$D_0 = \mathrm{Tr}\left[\sum_{i=q}^{2q-1} h^i(\overline{P}) - \sum_{i=0}^{q-1} h^i(\overline{P})\right] > 0,$$

$$D_\infty = \mathrm{Tr}\left[\sum_{i=q}^{2q-1} h^i(\overline{P}) - \sum_{i=0}^{q-1} h^i\left(h^{2q}(\overline{P})\right)\right] < 0.$$

Hence there is a unique $\delta_{\max}$ such that $D_{\delta_{\max}} = 0$, which corresponds to that $\delta_{\max}$ satisfies (24). Furthermore, for all $\delta < \delta_{\max}$, $D_\delta > 0$ and for all $\delta > \delta_{\max}$, $D_\delta < 0$. From part 3 of Lemma 4.4, for any $\delta < \sqrt{\lambda_{\min}\left(h^q(\overline{P}) - \overline{P}\right)}$,

$$D_\delta = \text{Tr} \left[\sum_{i=q}^{2q-1} h^i(\overline{P}) - \sum_{i=0}^{q-1} h^i(\overline{P} + \Delta_\delta) \right]$$

$$\leq \text{Tr} \left[\sum_{i=q}^{2q-1} h^i(\overline{P}) - \sum_{i=0}^{q-1} h^i(\overline{P} + \delta^2 I) \right]$$

$$< \text{Tr} \left[\sum_{i=q}^{2q-1} h^i(\overline{P}) - \sum_{i=0}^{q-1} h^i(\overline{P} + h^q(\overline{P}) - \overline{P}) \right]$$

$$= 0.$$

Therefore $\sqrt{\lambda_{\min}\left(h^q(\overline{P}) - \overline{P}\right)} \leq \delta_{\max}$. (2) Let $\delta_{\max} > 0$. For any $\delta \in (0, \delta_{\max})$ and for any realization ϕ of θ_h, if $|\varepsilon_{2lq}| > \delta$ for all odd number l, then ϕ is the same as θ_t^*. Hence $J(\phi) = J(\theta_t^*)$. Otherwise if there exists an odd number l such that $|\varepsilon_{2lq}| \leq \delta$, then similar to the proof of the first statement, one easily verifies $J(\phi) < J(\theta_t^*)$. Notice that the probability of those ϕ's with at least one l such that $|\varepsilon_{2lq}| \leq \delta$ is positive, hence

$$J(\theta_h) = \sum_\phi \Pr(\phi) J(\phi) < \sum_\phi \Pr(\phi) J(\theta_t^*) = J(\theta_t^*). \quad \blacksquare$$

To maximize the difference between $J(\theta_h)$ and $J(\theta_t^*)$, we simply pick up the δ which maximizes $\Pr\left(|\varepsilon_{2lq}| \leq \delta\right) D_\delta$. Finding the closed-form expression of the optimal δ is in general difficult. Nevertheless, it can be obtained from simple numerical tools such as a Monte Carlo simulation.

5. EXAMPLE

Consider the following parameters for system (1-2): $A = 1.01, C = 1, Q = R = 0.5, m = 99, T = 399$. The optimal time-based schedule θ_t^* is periodic with period 4. Fig. 3 plots $J(\theta_t^*)$ and $J(\theta_h(\delta))$ for different values of δ, which clearly demonstrates Theorem 4.5. From the figure, $\delta_{\max}$ equals 2.002, and the optimal δ equals 1 where the difference between $J(\theta_h(\delta))$ and $J(\theta_t^*)$ achieves its maximum.

6. CONCLUSION

In this paper, we present a joint time-based and event-based schedule to tackle the problem of remote state estimation with limited sensor communications. This novel schedule leads to better performance when compared with time-based schedules and does not require much computation resource when compared with event-based schedules. Future work include extensions to closed-loop control data scheduling and multiple sensor scheduling.

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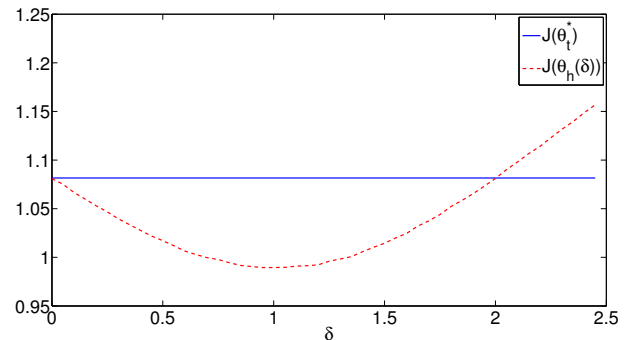
Fig. 3. Comparison of θ_h and θ_t^*

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