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# Tracking Performance Limitations in a Linear Feedback System with Remote Sensors ⋆

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**Abstract:** This paper studies tracking performance limitations for a networked feedback control system. In the system, the plant is a linear time invariant (LTI) SISO system and the measurement signal is received from a remote site through a network. The reference signal in the tracking problem is a step signal. The tracking performance is measured by an integral square error between the output of the plant and the reference signal. To transmit the measurement signal through a network, this signal is quantized and then certain information which the original signal possesses could be lost. The major issue which we study in this paper is: How does a logarithmic quantization law constrain the best attainable tracking performance of the feedback system? Here the quantization error is modeled as a product of the original signal and a bounded nonlinear function. An upper bound of the best attainable tracking performance of the system is presented in terms of the quantization error model and the characteristics of the plant. It is also found that, if the nonlinear function in the quantization error model is an $\mathcal{H}_\infty$ norm-bounded uncertainty, this upper bound is the tracking performance limit of the feedback system under the worst uncertainty. In the case where the quantizer and network are not used in the system, the upper bound is equal to the tracking performance limit of the LTI system.

## 1. INTRODUCTION

In this paper, tracking performance limitations are considered for a linear time invariant (LTI) SISO system with a quantized measurement. The reference signal of the LTI system is a step signal. The tracking performance is measured by the integral of the square tracking error between the reference and the output of the system. Due to the fact that the measurement signal would lose certain information after quantization, the performance of the networked system would deteriorate. Our major propose is to study: How does quantization error degrade the performance of the system and what is the fundamental limitation on the attainable optimal tracking performance for all possible controllers in terms of the features of the quantizer and the characteristics and structure of the plant.

The tracking problem for an LTI system with a quantized measurement includes two tasks: 1) to asymptotically track the reference signal and 2) to attenuate the disturbance caused by the measurement quantization. The sys-

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tem under consideration has a sensor in a remote site. It is assumed that the measurement is the tracking error which is quantized by a logarithmic quantization law. In many applications this is reasonable since the set point is known. For the case where the set point is unknown, the step magnitude could be sent to the measurer though network. The advantage of this scheme is that the asymptotical tracking is achievable. The error generated by the quantization law is a product of the original and a uniform bounded nonlinear function, referred to as relative quantization error in this paper. An upper bound of tracking performance limit is obtained in terms of the characteristics of the plant and the upper bound on the relative quantization error for the system. It is shown that this tracking performance upper bound is equal to the tracking performance limit of the system in the case where the quantization error approaches zero. Subsequently, we consider the case when the relative quantization error is modeled as an $\mathcal{H}_\infty$ norm-bounded uncertainty. In this case, the best attainable tracking performance of the system under the worst uncertainty is equal to the upper bound on the tracking performance limit obtained for the networked system.

Networked feedback system has attracted many research works (see Braslavsky et al. [2007], Elia and Mitter [2001], Fu and Xie [2005], Nair and Evans [2004] and the references therein) in past several years. A major problem on which existing work has focused is the stabilization of networked systems. In Elia and Mitter [2001] and Fu and Xie [2005], the stabilization problem is studied for a networked feedback system with a quantized control signal. The maximum relative quantization error of a logarithmic quantization law, which is allowed in the stabilization problem, is obtained in terms of the characteristics of the plant in the feedback system. Nair el. at. (see Nair and Evans [2004]) discussed the stabilization problem for networked systems with a communication channel data rate constraint. They presented the minimum channel data rate for the system in order to stabilize a given plant. In Braslavsky et al. [2007], the stabilization problem is discussed for networked systems under a signal-to-noise ratio constraint model of communication channels. Lower bounds of signal-to-noise ratio in the communication channel are given for state feedback and output feedback stabilization problems respectively. On the other hand, the study of performance limitations is an open area for networked feedback systems. This problem has widely been studied for LTI systems during last three decades. It was shown in Kwakernaak and Sivan [1972] that, for a square invertible LTI minimum phase systems, the optimal control performance limit is zero. Since then, the research on this issue was extended to nonminimum phase systems in (Morari and Zafiriou [1989], Qiu and Davison [1993], Chen et al. [2000] and Su, et al. [2003, 2007]). These works show that the performance limits for optimal LQR or optimal tracking problems is only dependent on the nonminimum phase zeros and the directional vectors associated with these zeros as well as the initial state or the reference signal. On the other hand, some efforts have also been made on this issue for an LTI system with disturbance input Davison and Scherzinger [1987], Jemaa and Davison [2003] and Xie and Petersen [2002]. It was shown by Davison and Scherzinger that, for a minimum phase LTI system with disturbance inputs, the robust performance limit is zero in Davison and Scherzinger [1987]. This discussion was also extended to nonminimum phase systems recently in Jemaa and Davison [2003]. Some interesting discussion on robust performance limitations for an LTI system with an uncertainty was presented in Goodwin et al. [2003].

Finally, a note on the notation: A signal in the discrete-time domain is denoted by a lower case letter, such as $r(l)$, and $l$ will be omitted without confusion. A system, viewed as an input/output operator, is denoted by a capital letter, such as $G$. The Laplace transform is denoted by a hat "$\hat{\ }$", i.e., $\hat{r}$ is the Laplace transform of $r$. If $G$ is LTI, $\hat{G}$ represents the transfer function of $G$. For any complex number, vector and matrix, denote their conjugate, transpose, conjugate transpose, real and imaginary part by $\bar{(\cdot)}$, $(\cdot)'$, $(\cdot)^*$, $\mathrm{Re}(\cdot)$ and $\mathrm{Im}\,(\cdot)$, respectively. The argument of a nonzero complex number is denoted by $\angle(\cdot)$. Let the open unit disk and the unit circle be denoted by $\mathbb{D}$ and $\mathbb{T}$, respectively. The usual Lebesgue space of vector-valued square integrable functions on $\mathbb{T}$ is denoted by $\mathcal{L}_2$. The set of those functions in $\mathcal{L}_2$ which are analytic in $\mathbb{D}$ is denoted by $\mathcal{H}_2$ and the set of those in $\mathcal{L}_2$ that are analytic on the complement of $\mathbb{D} \cup \mathbb{T}$ and vanish at the origin is denoted by $\mathcal{H}_2^\perp$. It is well-known that $\mathcal{H}_2$ and $\mathcal{H}_2^\perp$ form orthogonal complements as subspaces of $\mathcal{L}_2$. The Euclidean vector norm and the norm in the space $\mathcal{L}_2$ are both denoted by $\|\cdot\|_2$. The symbol $\mathcal{RH}_\infty$ denotes the set of all stable, rational transfer matrices. Finally, the inner product of two complex vectors $u, v$ is defined as $\langle u, v\rangle := u^* v$.

## 2. PRELIMINARIES

Suppose $G$ be a given SISO plant in the feedback system shown in Figure 1. The transfer function of the plant is a proper rational function $\hat{G}(\lambda)$ where $\lambda$ is the unit delay operator. In the system, $K_1$ and $K_2$ make up the 2 degree of freedom (2DOF) controller to be designed. The signals $r$ and $z$ are a step reference signal with a magnitude $v$ and the output of the system, respectively. It is assumed that $\hat{G}(\lambda)$ is right-invertible. Let $\hat{N}(\lambda)\hat{D}^{-1}(\lambda)$, $\hat{D}(\lambda)$, $\hat{N}(\lambda) \in \mathcal{R}H_\infty$, and $\tilde{\hat{D}}^{-1}(\lambda)\tilde{\hat{N}}(\lambda)$, $\tilde{\hat{D}}(\lambda)$, $\tilde{\hat{N}}(\lambda) \in \mathcal{R}H_\infty$ be right and left coprime factorizations of $\hat{G}$, respectively. Then, there exist $\hat{X}(\lambda), \hat{Y}(\lambda), \tilde{\hat{X}}(\lambda), \tilde{\hat{Y}}(\lambda) \in \mathcal{R}H_\infty$ satisfying the double Bezout identity (see Vidyasagar [1985])

$$\begin{bmatrix} \tilde{\hat{X}}(\lambda) & -\tilde{\hat{Y}}(\lambda) \\ -\hat{\tilde{N}}(\lambda) & \tilde{\hat{D}}(\lambda) \end{bmatrix} \begin{bmatrix} \hat{D}(\lambda) & \hat{Y}(\lambda) \\ \hat{N}(\lambda) & \hat{X}(\lambda) \end{bmatrix} = I. \tag{1}$$

The set of 2DOF controllers stabilizing $G$ is given by

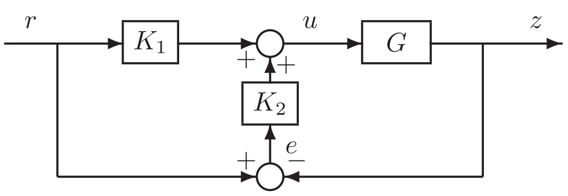


Fig. 1. A feedback system with a two degree freedom controller

$$\mathcal{K} = \Big\{[K_1\ K_2] = (\tilde{X} - R\tilde{N})^{-1}[Q - (\tilde{Y} - R\tilde{D})\ \ (\tilde{Y} - R\tilde{D})],$$
$$Q, R \in \mathcal{R}H_\infty, \ \ |\tilde{X} - R\tilde{N}| \neq 0\Big\} \tag{2}$$

where $Q$ and $R$ are causal stable factors to be designed. The set $\mathcal{K}$ includes all possible controller stabilizing the feedback system.

Suppose that $z_i \in \mathbb{T}$, $i = 1, \cdots, m$ are zeros of the nonminimum phase plane $\hat{G}(\lambda)$. The transfer function $\hat{G}(\lambda)$ can be factorized as follows:

$$\hat{G}(\lambda) = \hat{G}_{in}(\lambda)\hat{G}_o(\lambda) \tag{3}$$

and

$$\hat{G}_{in}(\lambda) = \prod_{i=1}^{k} \hat{G}_i(\lambda) \tag{4}$$

where $\hat{G}_i(\lambda)$ is an inner with only one zero $z_i$,

$$\hat{G}_i(\lambda) = \frac{1-\bar{z}_i}{1-z_i}\frac{\lambda - z_i}{1-\bar{z}_i\lambda}$$

and $\hat{G}_0(\lambda)$ has no nonminimum phase zero. A standard form of $\hat{G}_i(\lambda)$, referred a Blaschke factor in the literature. $\hat{G}_{in}(\lambda)$ and $\hat{G}_o$ are called an *inner* and *outer*, respectively. The factorization $\hat{G}(\lambda) = \hat{G}_{in}(\lambda)\hat{G}_o(\lambda)$ is called the *inner-outer factorization.*

In the remainder of this section, we introduce a logarithmic quantization law (see e.g. Elia and Mitter [2001] and Fu and Xie [2005]) which is used in the networked system under consideration. Let $q$ be a positive constant and $0 < q < 1$. The logarithmic quantization law is defined as follows:

$$e_q = \begin{cases} sgn(e)q^i\,, & \text{if } \frac{1}{2}(1+q)q^i < |e| \leq \frac{1}{2}(1+q)q^{i-1} \\ 0\,, & \text{if } e = 0 \end{cases} \tag{5}$$

where sgn$(\cdot)$ is the sign function.

Denote the quantization error, which is the difference between the input and the output of the quantizer, by $d_q$, i.e.,

$$d_q = e - e_q. \tag{6}$$

Define the relative quantization error:

$$\Delta_q = \frac{d_q}{e}. \tag{7}$$

It is clear from the definition of the logarithmic quantization law (5) that

$$|\Delta_q| \leq \delta_q, \qquad \text{and} \qquad \delta_q = \frac{1-q}{1+q}.$$

## 3. ASYMPTOTICALLY TRACKING PROBLEM OF A NETWORKED SYSTEM

We now formulate an asymptotic tracking problem for the feedback control system shown in Figure 1 in which a part of the communication is implemented through a network as shown in Figure 2. The goals of this problem are to internally stabilize the closed-loop system and to achieve asymptotic tracking for a step reference signal.

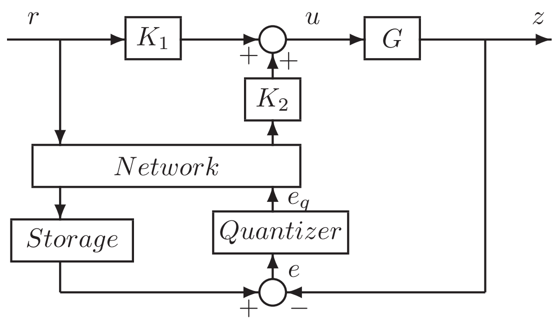


Fig. 2. Networked tracking system with a quantizer

The sensor of the system is in a remote site, the measured signal is quantized and sent to the controller through a network. The quantization law used in the system is a logarithmic quantization law (see e.g. Elia and Mitter [2001], Fu and Xie [2005]). To achieve asymptotic tracking, the magnitude $v$ of the step signal is sent to the remote site and stored. Then the tracking error $e$, the difference between the reference and the output of the system, is quantized and sent to the controller of the system through the network. This scheme guarantees that the quantization error approaches zero as the tracking error vanishes. The signal $e_q$ is the output of the quantizer.

The tracking performance is measured by the energy of the tracking error $e = r - z$:

$$J(v) = \sum_{l=0}^{\infty} e^2(l). \tag{8}$$

Notice the fact that the quantizer removes some information which the feedback signal $e$ possesses. Hence the stability robustness and performance of the system are degraded. The major problems which we are interested in are: How does the quantization error affect the tracking performance of the networked system? What is the best attainable performance of the networked feedback system in tracking a step reference signal? This best attainable performance is referred to as the *tracking performance limit* which is the minimal value of $J(v)$ in (8) over all possible controllers in $\mathcal{K}$:

$$J_{n.opt}(v) = \inf_{[K_1 \;\; K_2]\in\mathcal{K}} J(v). \tag{9}$$

## 4. TRACKING PERFORMANCE LIMIT OF THE NETWORKED SYSTEM

Consider the tracking problem for the system shown in Figure 2. In order for the problem to be meaningful and solvable, we make the following assumption throughout the paper.

*Assumption 4.1.* $\hat{G}(\lambda)$ has no zero at the origin.

Following (6) and (7), we have

$$e_q = e + d_q \qquad \text{and} \qquad d_q = \Delta_q e. \tag{10}$$

The networked system in Figure 2 is formulated as an LTI system with a bounded nonlinear component $\Delta_q$ as shown in Figure 3. Applying the 2DOF controller in (2) to this

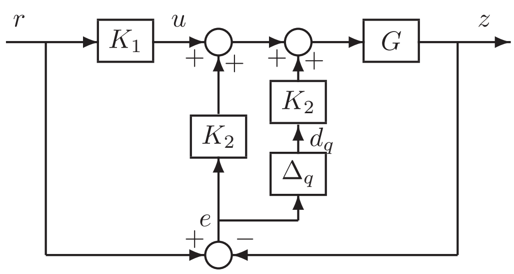


Fig. 3. Tracking system with a quantizer

system leads to:

$$e = (I - NQ)r + T_{zq}\Delta_q e \tag{11}$$

where

$$T_{zq} = N(\tilde{X} - R\tilde{N}).$$

*Theorem 4.1.* Let $\hat{G}$ have nonminimum phase zeros $z_1, z_2, \ldots, z_m$. Then the tracking performance limit of the system is given by

$$J_{n.opt}(v) \le \frac{2}{1 - \delta_q^2 \inf_R \|T_{zq}\|_\infty^2} \sum_{i=1}^{m} \frac{\mathrm{Re}\,(z_i)}{|z_i|^2}. \tag{12}$$

and

$$\inf_R \|T_{zq}\|_\infty < \frac{1}{\delta_q}. \tag{13}$$

The proof is given in Appendix A.

Theorem 4.1 gives an upper bound for the tracking performance limit of the networked LTI system. In particular, this upper bound is exactly equal to the tracking performance limit of the system (see Chen et al. [2000]) in the case where the controller in the feedback system accesses the complete information of the measurement signal, i.e., $\delta_q = 0$.

## 5. TRACKING PERFORMANCE LIMITATION OF AN UNCERTAIN SYSTEM

In this section, we present a tracking performance limit of the system shown in Figure 3 for the case where the component $\Delta_q$ is an $\mathcal{H}_\infty$ norm-bounded uncertainty, i.e.,

$$\|\Delta_q\|_\infty \le \delta_q. \tag{14}$$

Compared with the relative quantization error $\Delta_q$ which is studied in last section, the uncertainty in (14) includes more general types of components which could be a nonlinear time-invariant uncertainty or linear time varying uncertainty.

Here the performance limit to be studied is the best attainable performance of the system over all possible controllers from $\mathcal{K}$ for the worst case uncertainty in (14). It is referred as the *minimax asymptotic tracking/robust stabilization performance limit*:

$$J_{r.opt}(v) = \inf_{[K_1 \;\; K_2]\in\mathcal{K}} \sup_{\|\Delta_q\|_\infty \le \delta_q} J(v). \tag{15}$$

It still is assumed that Assumption 4.1 is satisfied.

*Theorem 5.1.* Let $\hat{G}$ have nonminimum phase zeros $z_1, z_2, \ldots, z_m$ and suppose the uncertainty satisfies (14). Then the minimax asymptotic tracking/robust stabilization performance limit of the system shown Figure 3 is given by

$$J_{r.opt}(v) = \frac{2}{1 - \delta_q^2 \inf_R \|T_{zq}\|_\infty^2} \sum_{i=1}^{m} \frac{\mathrm{Re}\,(z_i)}{|z_i|^2}. \tag{16}$$

Moreover, the worst case uncertainty could be nonlinear time invariant or linear time varying.

The proof is omitted.

This formula shows that the upper bound of the tracking performance limit of the networked system given in last section is the minimax asymptotic tracking/robust stabilization performance limit of the system with an $\mathcal{H}_\infty$ norm bounded uncertainty. This follows from the fact that the relative quantization error studied in the last section is one of the norm bounded functionals given by (14) but may not be the worst one among the uncertainties given by (14).

The result in this theorem shows that the performance limit $J_{r.opt}(v)$ in (16) is related to the tracking performance limit of the system without uncertainty (see Chen et al. [2000]) and the lowest achievable closed-loop gain in the system. The tracking performance limit is achieved by selecting the parameter $Q$ in a 2DOF controller while the lowest achievable closed-loop gain of the system is only related to the parameter $R$ in a 2DOF controller, i.e.,

$$J_{r.opt}(v) = \frac{\inf\limits_{Q} \|(I - NQ)r\|_2^2}{1 - \delta_q^2 \inf\limits_{R} \|T_{zq}\|_\infty^2}. \tag{17}$$

From these results, a sub-optimal tracking problem for the network system can be decomposed into two independent simpler optimal controller design problems: optimal tracking problem and optimal robust stabilization problem.

## 6. CONCLUSION

In this paper, we discussed tracking performance limitations for a networked feedback system. In the system, the measurement is quantized by a logarithmic quantization law. A relative quantization error is modeled as a bounded nonlinear function and an upper bound of the tracking performance limit is obtained for the networked feedback system. It is shown that, if the relative quantization error under consideration in the networked system is an $\mathcal{H}_\infty$ norm-bounded uncertainty, the tracking performance limit of the system under the worst uncertainty is equal to the upper bound of tracking performance limit which we obtained for the networked feedback system. In addition, the worst uncertainty could be a nonlinear time invariant or linear time varying component. In these results, we also see that the sub-optimal tracking problem for the networked system can be decomposed into two simpler problems: One is the optimal tracking problem for the system without the quantizer and the other is the optimal robust stabilization problem for an LTI system with $\mathcal{H}_\infty$ norm-bounded uncertainty.

## APPENDIX A

## Proof of Theorem 4.1

To prove this theorem, we will consider an asymptotic tracking/disturbance rejection problem for the LTI system shown in Figure 1 in the case where it is disturbed by a signal $d$, as shown in Figure 4. It is assumed that the disturbance signal $d$ is from $\mathcal{L}_2$, i.e.,

$$\sum_{l=0}^{\infty} d^2(l) < \infty. \tag{A-1}$$

$W$ is a known LTI stable, proper weight. The asymptotic

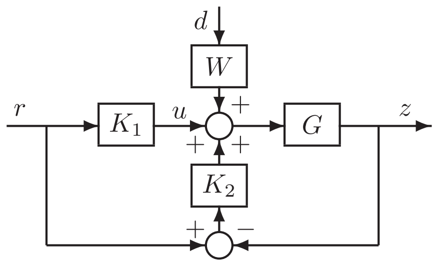


Fig. 4. The LTI system in the regulation/disturbance problem

tracking/disturbance rejection problem is to design a controller $[K_1\ K_2]$ so that the closed loop system is internally stabilized and the plant output $z$ asymptotically tracks the step signal $r$ for any disturbance input $d$ from $\mathcal{L}_2$. The tracking performance is measured by the energy of the tracking error in (8). The best attainable tracking performance of the system by all possible controllers from $\mathcal{K}$ given by (2) under the worst disturbance input is considered. This performance limit is referred as *minimax asymptotic tracking/disturbance rejection performance limit*, i.e.,

$$J_{d.opt} = \inf_{[K_1\ K_2] \in \mathcal{K}_0} \sup_{d \in \mathcal{L}_2} J(v). \tag{A-2}$$

It is assumed that the plant $G$ satisfies Assumption 4.1 and denote the operator from disturbance input $d$ and output $z$ by $T_{zd}$.

For the disturbance free case, this performance limit is given in Chen et al. [2000].

*Lemma A.1.* Suppose that the system shown in Figure 4 satisfies Assumption 4.1, the reference signal $r$ is given by a step signal $r(l) = v$, $l = \{0, 1, 2, \cdots, \infty\}$, and the disturbance input $d(l) \equiv 0$. Let $\hat{G}$ have nonminimum phase zeros $z_1, z_2, \ldots, z_m$. $\hat{G}_{in}$ is the inner factor of $\hat{G}$ given by (4). The tracking performance limit is given by

$$J_{opt}(v) = \inf_{[K_1 \;\; K_2]\in\mathcal{K}_0} J(v) = \|\hat{r} - \hat{G}_{in}\hat{r}\|_2^2 = 2\sum_{i=1}^{m} \frac{\mathrm{Re}\,(z_i)}{|z_i|^2}.$$

In general, the minimax asymptotic tracking/disturbance rejection performance limit of the system is presented in the following lemma.

*Lemma A.2.* Let $\hat{G}$ have nonminimum phase zeros $z_1, z_2$, $\ldots$, $z_m$. Then the minimax asymptotic tracking/disturbance rejection performance limit of the system is given by

$$J_{d.opt}(v) = 2\sum_{i=1}^{m} \frac{\mathrm{Re}\,(z_i)}{|z_i|^2} + \inf_{R} \|T_{zd}\|_\infty^2 \|d\|_2^2. \qquad \text{(A-3)}$$

The proof is omitted.

*Proof of Theorem 4.1*

Let the disturbance input $d$ in the system in Figure 4 be the quantization error $d_q$ of the system shown in Figure 3. Apply any 2DOF controller from $\mathcal{K}$ to the system in Figure 4 and select $W = K_2$. The system in Figure 4 is transferred into the system in Figure 3 and it holds that

$$T_{zd} = T_{zq}.$$

Following from the proof of Lemma A.2, it is straightforward to obtain that

$$\|e\|_2^2 = \|\tilde{r}\|_2^2 + \|T_{zq}d_q\| = \|\tilde{r}\|_2^2 + \|T_{zq}\Delta_q e\|_2^2 \qquad \text{(A-4)}$$

where $\tilde{r} = r - G_{in}r$.

Subsequently, from the definition of $\mathcal{H}_\infty$ norm and the upper bound $\delta_q$ of the relative quantization error $\Delta_q$, the equation (A-4) is written as

$$\|e\|_2^2 \leq \|\tilde{r}\|_2^2 + \|T_{zq}\|_\infty^2 \delta_q^2 \|e\|_2^2. \qquad \text{(A-5)}$$

Then, using Lemma A.1, we have

$$\begin{aligned}\|e\|_2^2 &\leq \frac{\|\tilde{r}\|_2^2}{1 - \|T_{zq}\|_\infty^2 \delta_q^2} \\ &= \frac{2}{1 - \|T_{zq}\|_\infty^2 \delta_q^2} \sum_{i=1}^{m} \frac{\mathrm{Re}\,(z_i)}{|z_i|^2}. \qquad \text{(A-6)}\end{aligned}$$

Consequently, the inequality (12) holds.

On the other hand, it follows from the Small Gain Theorem (see for example Zhou et al. [1995]) and (11) that the inequality (13) guarantees the stability of the system.

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