14th Triennial World Congress, Beijing, P.R. China

D-2b-06-1

MODEL PREDICTIVE CONTROL FOR SAMPLED-DATA SYSTEMS

Li Chai*, Yong-Yan Cao, You-Xian Sun** and Li Qiu***

*Department of Electrical and Electronic Engineering
Hong Kong University of Science and Technology
Clear Water Bay, Kowloon
Hong Kong

**Institute for Industrial Process Control
Zhejiang University
Hangzhou, P. R. China

Abstract: In this paper, the model predictive control law for sampled-data systems is presented. Compared to the standard model predictive controller, the new controller considers the intersampling behavior of the continuous system. First we convert the hybrid optimization problem to an equivalent discrete-time one and solve it using dynamic programming. Then we show that the controller is stabilizing. *Copyright © 1999 IFAC*

Key Words: sampled-data systems, model predictive control, hybrid systems.

1 INTRODUCTION

Recently, model predictive control (MPC), also known as receding horizon control and moving horizon control, has become a popular technique to control systems and achieved a significant level of industrial success in practical process control applications. The main idea of MPC is as follows. At sampling time k, the M (called input horizon) future control moves that optimize the open-loop performance objective over some output horizon are calculated. Only the first one of the M computed control moves is implemented. At the next sampling time, the optimization problem is reformulated and solved with new measurements obtained from the system. Now, a great deal of results on stability (Clarke and Scattolini, 1991; Rawlings and Muske, 1993; Kouvaritakis et al., 1992; Zheng and Morari, 1995), robustness (Zafiriou, 1990; Gencelli and Nikolaou 1993; Zheng and Morari, 1993; Kothare, et al., 1996), and applications (Richalet, 1993; Camaco and Bordons, 1995) of the model predictive control have been presented. A fairly complete discussion of several design techniques based on MPC and the current state of model predictive control can be found in the review articles (Garcial et al., 1989, Kwon, 1994).

With the great advantages in computer technology, today digital controllers are more compact, reliable, flexible and often less expensive than analog ones. Note that significant progress has been made in modern analysis and synthesis for sampled-data feedback systems, where a continuous-time plant is controlled by a digital controller with appropriate hold and sample devices. By taking into account of the intersample behaviors, a sampled-data feedback control system combines both continuous and discrete-time dynamic subsystems. So it is a hybrid system with hybrid input and output signals. Although both the plant and controller are time-invariant, the sampled-data system is a time-varying system. Many researchers have investigated various problems in sampled-data control, such as the H_∞, H_2 and robust control problems (Bamieh and Pearson, 1992a, 1992b; Kabamba and Hara, 1993; Chen and Francis, 1991, 1996; Toivonen, 1992; Sun et al. 1993; Sivahsankar and Khargonekar, 1993, 1994; Hayakawa et al. 1994; Khargonekar and Sivahsankar, 1991; Hagiwara and Araki, 1995; Khammash, 1993; Dullerud, 1993, 1996). More complete discussions can be found in Hara et al. (1996) and Chen and Frances (1995).

 ISBN: 0 08 043248 4

Although MPC has many advantages, it usually uses a discrete time model only based on the behavior at the sampling instants. So it ignores the intersample behavior. To obtain a high performance digital controller for a continuous-time system, it is necessary to consider the hybrid nature of the problem. That is, the design should be a direct sampled-data design. The main contribution of this paper is that it presents a technique of designing a digital model predictive controller directly for sampled-data systems. The paper is organized as follows. In Section 2, we discuss some background material about sampled-data feedback systems and infinite horizon MPC. In Section 3 and 4, we formulate the control law of the MPC for sampled-data systems and show that the control law we obtain is stabilizing. Finally, in Section 5, the paper is concluded.

2 BACKGROUND

2.1 Sampled-Data Feedback Control Systems

First, we give a brief introduction to sampled-data systems. For more details, we refer the reader to Chen and Francis (1995). A typical sampled-data feedback control system is shown in Fig.1. Let us note the following points:

- $u(t)$, $y(t)$, $e(t)$, $r(t)$, $d(t)$ are continuous-time signals of the control input, plant output, reference command, tracking error and disturbance input. Continuous-time signals will be represented by () around an independent variable, whereas discrete-time signals will be represented by bracket [].
- $u[k]$, $y[k]$, $e[k]$, $n[k]$ are digital signals of the output of the controller, error at sampling instants and measurement noise, respectively.
- Let h be the sampling period. S and H are the ideal sampler and hold operator respectively. That is

$$Se(t) = e(kh) = e[k];\ k = 0, 1, 2, \ldots$$

$$(\mathrm{H}u)(kh+\theta) = \mathrm{H}(\theta)u[k] = \mathrm{u}(kh+\theta);\ 0 < \theta \leq h.$$

In this paper, we use the usual zero order hold, i.e. $\mathrm{H}(\theta) = I$.

The purpose of the sampled-data control design is to find a stabilizing digital controller which gives desirable property. In the control design, the error e(t) rather than the e[k] is considered. In other words, we have to take into account the intersampling behavior in sampled-data design.

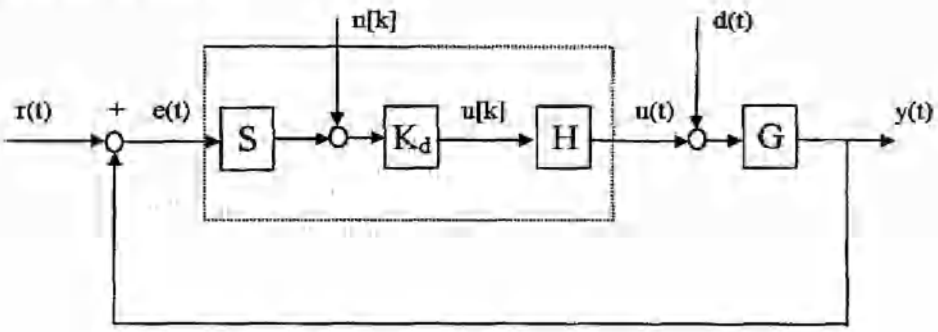

Figure 1: A Typical Sampled-data Feedback System

2.2 Model predictive control

Consider the discrete-time linear system

$$\mathbf{x}[k+1] = \mathbf{A}_d\mathbf{x}[k] + \mathbf{B}_d\mathbf{u}[k],$$

$$\mathbf{y}[k] = \mathbf{C}_d\mathbf{x}[k]$$

where $\mathbf{u}[k]$蜉^m is the control input, $\mathbf{x}[k]$蜉^n is the state of the plant and $\mathbf{y}[k]$蜉^p is the plant output. At each sampling time k we determine $\mathbf{u}[k+i|k]$, i=0,...,M-1, by minimizing the objective function $J_P[k]$, subject to some constraints on the control input, and sometimes also on the state or output, where $J_P[k]$ is a given norm presentation related to $\mathbf{x}[k+i|k]$, $i = 0, \ldots, P-1$, and $\mathbf{u}[k+i|k]$, $i = 0, \ldots, M-1$, ($\mathbf{y}[k+i|k]$ can be expressed by $\mathbf{x}[k+i|k]$). Here the notation is general:

$\mathbf{x}[k+i|k]$ = predicted value of the state at time $k+i$ based on the information available at time k; $\mathbf{x}[k|k]$ refers to the state measured at time k

$\mathbf{u}[k+i|k]$ = control input at time $k+i$, computed by the optimization problem at time k; $\mathbf{u}[k|k]$ is the control input to be implemented at time k

P = prediction horizon

M = control horizon

In the model predictive framework, only the first computed control move $\mathbf{u}[k|k]$ is implemented. At time k+1, the optimization is resolved with new measurements from the plant. The purpose of taking new measurements at each time step is to compensate for unmeasured disturbances and

ISBN: 0 08 043248 4

model uncertainty. This is the main feature of the model predictive control.

3 MPC FOR SAMPLED-DATA SYSTEMS

Let ($\mathbf{A}_c$, $\mathbf{B}_c$, $\mathbf{C}_c$) be a given continuous-time system

$$\begin{aligned} &\dot{\mathbf{x}}(t) = \mathbf{A}_c\mathbf{x}(t) + \mathbf{B}_c\mathbf{u}(t), \\ &\mathbf{y}(t) = \mathbf{C}_c\mathbf{x}(t), \\ &\mathbf{x}(0) = \mathbf{x}_0, \quad \mathbf{u} \in \mathbf{R}^m, \quad \mathbf{x} \in \mathbf{R}^n, \quad \mathbf{y} \in \mathbf{R}^p \end{aligned} \tag{1}$$

Let h be the sampling period. At each sampling time kh, assume that exact measurement of the state of the system is available, i.e., $\mathbf{x}(kh|kh) = \mathbf{x}(kh)$, and Mh is the control horizon, i.e.

$$\mathbf{u}(kh+t|kh) = 0, \ \forall t > Mh.$$

The problem is to design the control input $\mathbf{u}(kh+t|kh)$, $0 \le t \le Mh$, such that the following quadratic objective function

$$\min_{\mathbf{u}(kh+t|kh), 0\le t\le Mh} J = \int_{kh}^{\infty}\left[\mathbf{x}^T(t)\mathbf{Q}\mathbf{x}(t) + \mathbf{u}^T(t)\mathbf{R}\mathbf{u}(t)\right]dt$$

is minimized, where $\mathbf{Q}$ and $\mathbf{R}$ are positive definite, symmetric weighting matrices. Here we use infinite horizon optimization to guarantee the nominal stability. Using the zero order hold, the objective function can be rewritten as

$$\min_{\mathbf{u}[k+i|k], i=0,\cdots,M-1} J = \int_{kh}^{\infty}\left[\mathbf{x}^T(t)\mathbf{Q}\mathbf{x}(t) + \mathbf{u}^T(t)\mathbf{R}\mathbf{u}(t)\right]dt \tag{2}$$

Note that this is a hybrid optimization problem since both the continuous-time and discrete-time signals are involved in (2).

The discretized system of ($\mathbf{A}_c$, $\mathbf{B}_c$, $\mathbf{C}_c$) is as follows

$$\begin{aligned} \mathbf{x}[k+1] &= \mathbf{A}_d\mathbf{x}[k] + \mathbf{B}_d\mathbf{u}[k], \\ \mathbf{y}[k] &= \mathbf{C}_c\mathbf{x}[k] \end{aligned} \tag{3}$$

where

$$\mathbf{x}[k] = \mathbf{x}(kh), \ \mathbf{y}[k] = \mathbf{y}(kh),$$

$$\mathbf{A}_d = e^{\mathbf{A}h}, \quad \mathbf{B}_d = \left(\int_0^h e^{\mathbf{A}t}dt\right)\mathbf{B}$$

From (1) and (3), we have

$$\mathbf{x}(kh+t) = e^{\mathbf{A}t}\mathbf{x}[k] + \int_0^t e^{\mathbf{A}(t-\tau)}d\tau\,\mathbf{B}\mathbf{u}[k], \tag{4}$$

where $0 \le t \le h$, Now we are ready to give the main result of this section.

Theorem 1 Consider the stable system ($\mathbf{A}_c$, $\mathbf{B}_c$, $\mathbf{C}_c$) in (1) together with the objective function (2). The optimal discrete state feedback controller can be obtained as follows

$$\mathbf{u}[k+i|k] = -\mathbf{F}[i]\mathbf{x}[k+i|k] \tag{5}$$

$$\mathbf{F}[i] = \left[\mathbf{Q}_{22} + \mathbf{A}_d^T\mathbf{S}[i+1]\mathbf{A}_d\right]^{-1}\left[\mathbf{A}_d^T\mathbf{S}[i+1]\mathbf{B}_d + \mathbf{Q}_{12}^T\right] \tag{6}$$

$$\begin{aligned} \mathbf{S}[i] &= (\mathbf{B}_d - \mathbf{A}_d\mathbf{F}[i])^T\mathbf{S}[i+1](\mathbf{B}_d - \mathbf{A}_d\mathbf{F}[i]) \\ &\quad + \mathbf{F}^T[i]\mathbf{Q}_{22}\mathbf{F}[i] + \mathbf{Q}_{11} - \mathbf{F}^T[i]\mathbf{Q}_{12}^T - \mathbf{Q}_{12}\mathbf{F}[i], \end{aligned} \tag{7}$$

$$i = 1, \cdots M-1$$

$$\mathbf{S}[M] = \overline{\mathbf{Q}} \tag{8}$$

where

$$\mathbf{Q}_{11} = \int_0^h e^{\mathbf{A}_c^T t}\mathbf{Q}e^{\mathbf{A}_c t}dt \tag{9}$$

$$\mathbf{Q}_{12} = \left[\int_0^h e^{\mathbf{A}_c^T t}\mathbf{Q}\left(\int_0^t e^{\mathbf{A}_c\tau}d\tau\right)dt\right]\mathbf{B}_c \tag{10}$$

$$\mathbf{Q}_{22} = \mathbf{B}_c^T\left[\int_0^h\left(\int_0^t e^{\mathbf{A}_c^T\tau}d\tau\right)\mathbf{Q}\left(\int_0^t e^{\mathbf{A}_c\tau}d\tau\right)dt\right]\mathbf{B}_c + h\mathbf{R} \tag{11}$$

$$\overline{\mathbf{Q}} = \int_0^{+\infty} e^{\mathbf{A}_c^T t}\mathbf{Q}e^{\mathbf{A}_c t}dt \tag{12}$$

Proof At first, we will convert the hybrid optimization problem to a discrete-time one, then use the dynamic program to obtain the solution. Since Mh is the control horizon, there is no control input after Mh sampling interval. For any $t > 0$, we have

$$\begin{aligned} &\mathbf{x}(Mh + kh + t|kh) \\ &= e^{\mathbf{A}_c(t-Mh)}\mathbf{x}(Mh + kh|kh) \\ &= e^{\mathbf{A}_c(t-Mh)}\mathbf{x}[M + k|k] \end{aligned} \tag{13}$$

The objective function can be rewritten as

$$\begin{aligned} J &= \int_{kh}^{(k+M)h}\left[\mathbf{x}^T(t)\mathbf{Q}\mathbf{x}(t) + \mathbf{u}^T(t)\mathbf{R}\mathbf{u}(t)\right]dt \\ &\quad + \int_{(k+M)h}^{+\infty}\mathbf{x}^T(t)\mathbf{Q}\mathbf{x}(t)dt \\ &= \sum_{i=0}^{M-1}\int_{(k+i)h}^{(k+i+1)h}\left[\mathbf{x}^T(t)\mathbf{Q}\mathbf{x}(t) + \mathbf{u}^T(t)\mathbf{R}\mathbf{u}(t)\right]dt \\ &\quad + \mathbf{x}^T[k+M|k]\left(\int_{Mh}^{+\infty}e^{\mathbf{A}^T(t-Mh)}\mathbf{Q}e^{\mathbf{A}(t-Mh)}dt\right)\mathbf{x}[k+M|k] \\ &= \sum_{i=0}^{M-1}J_i + \mathbf{x}^T[k+M|k]\overline{\mathbf{Q}}\mathbf{x}[k+M|k] \end{aligned}$$

where

$$\begin{aligned} J_i &= \mathbf{x}^T[k+i|k]\mathbf{Q}_{11}\mathbf{x}[k+i|k] \\ &\quad + 2\mathbf{x}^T[k+i|k]\mathbf{Q}_{12}\mathbf{u}[k+i|k] \\ &\quad + \mathbf{u}^T[k+i|k]\mathbf{Q}_{22}\mathbf{u}[k+i|k] \end{aligned}$$

The hybrid optimization problem (2) is now converted to a discrete-time one

$$\min_{\mathbf{u}[0],\cdots\mathbf{u}[M-1]} J = \sum_{i=0}^{M-1}J_i + \mathbf{x}^T[k+M|k]\overline{\mathbf{Q}}\mathbf{x}[k+M|k] \tag{14}$$

Using discrete dynamic programming, the result can be obtained. □

Note that the equivalent discrete objective function includes a crossover term which differs from the discrete MPC case.

ISBN: 0 08 043248 4

Remark 1. The formulas for computing the matrices $\mathbf{Q}_{11}$, $\mathbf{Q}_{12}$, $\mathbf{Q}_{22}$ in (9)-(11) can be done using matrix exponential as in Franklin and Powell (1980) and Chen and Francis (1995). We will give the explicit formulas for $\mathbf{Q}_{11}$, $\mathbf{Q}_{12}$, $\mathbf{Q}_{22}$ in the following.

Lemma 1 (Chen and Francis, 1995) Let $\mathbf{A}_{11}$ and $\mathbf{A}_{22}$ both be square and define

$$\begin{bmatrix} \mathbf{F}_{11}(t) & \mathbf{F}_{12}(t) \\ \mathbf{0} & \mathbf{F}_{22} \end{bmatrix} := \exp\left\{t\begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{0} & \mathbf{A}_{22} \end{bmatrix}\right\}, \quad t \geq 0.$$

Then

$$\mathbf{F}_{11}(t) = e^{t\mathbf{A}_{11}},$$
$$\mathbf{F}_{22}(t) = e^{t\mathbf{A}_{22}},$$
$$\mathbf{F}_{12}(t) = \int_0^t e^{(t-\tau)\mathbf{A}_{11}} \mathbf{A}_{12} e^{\tau \mathbf{A}_{22}} d\tau .$$

Corollary 1 For the system (1), Define

$$\begin{bmatrix} \mathbf{P}_{11} & \mathbf{P}_{12} \\ \mathbf{0} & \mathbf{P}_{22} \end{bmatrix} = \exp\left\{h\begin{bmatrix} -\begin{bmatrix} \mathbf{A}_c & \mathbf{B}_c \\ 0 & 0 \end{bmatrix}^T & \begin{bmatrix} \mathbf{Q} & \mathbf{0} \\ \mathbf{0} & \mathbf{R} \end{bmatrix} \\ \mathbf{0} & \begin{bmatrix} \mathbf{A}_c & \mathbf{B}_c \\ 0 & 0 \end{bmatrix} \end{bmatrix}\right\}$$

Then $\mathbf{Q}_{11}$, $\mathbf{Q}_{12}$ and $\mathbf{Q}_{22}$ in (9)-(11) can be computed via the following

$$\begin{bmatrix} \mathbf{Q}_{11} & \mathbf{Q}_{12} \\ \mathbf{Q}_{12}^T & \mathbf{Q}_{22} \end{bmatrix} = \mathbf{P}_{22}{}^T \mathbf{P}_{12} .$$

Proof For any matrix **A**, we know that

$$\int_0^t e^{A\tau} d\tau = [\mathbf{I} \quad \mathbf{0}] e^{\begin{bmatrix} A & 1 \\ 0 & 0 \end{bmatrix} t} \begin{bmatrix} \mathbf{I} \\ \mathbf{0} \end{bmatrix}.$$

It is then straightforward to obtain the corollary by Lemma 1. □

Remark 2. Integrate the following equation from 0 to ∞

$$\frac{d}{dt} e^{t\mathbf{A}_c^T} \mathbf{Q} e^{t\mathbf{A}_c} = \mathbf{A}_c{}^T e^{t\mathbf{A}_c^T} \mathbf{Q} e^{t\mathbf{A}_c} + e^{t\mathbf{A}_c^T} \mathbf{Q} e^{t\mathbf{A}_c} \mathbf{A}_c$$

Since $\exp(t\mathbf{A}_c)$ converges to zero ($\mathbf{A}_c$ is stable), we have

$$\mathbf{A}_c{}^T \overline{\mathbf{Q}} + \overline{\mathbf{Q}} \mathbf{A}_c + \mathbf{Q} = \mathbf{0} \tag{15}$$

Then $\overline{\mathbf{Q}}$ can be computed via the equation (15).

Remark 3. The MPC algorithm for sampled-data systems is then obtained from Theorem 1.

i. Compute $\mathbf{Q}_{11}$, $\mathbf{Q}_{12}$, $\mathbf{Q}_{22}$, $\overline{\mathbf{Q}}$ in (9-12). The explicit formulas of $\mathbf{Q}_{11}$, $\mathbf{Q}_{12}$, $\mathbf{Q}_{22}$, $\overline{\mathbf{Q}}$ is given in remark 1 and remark 2.
ii. Get the state measurement $\mathbf{x}(kh)$ at the sampling time kh, compute $\mathbf{S}[k]$, $\mathbf{F}[k]$ from (6)-(8) and compute the optimal control law from (5). Only the first element $\mathbf{u}[k|k]$ is sent to the process.
iii. Return to Step ii at the next sampling time $(k+1)h$.

4 STABILITY ANALYSIS

In this section, we will show that the above control law is stabilizing just like in the discrete-time case. To this end, we need the following lemma.

Lemma 2 Consider the system ($\mathbf{A}_c$, $\mathbf{B}_c$, $\mathbf{C}_c$) in (1) together with the objective function (2). Assume the sampling period h is not pathological, then the matrix $\begin{bmatrix} \mathbf{Q}_{11} & \mathbf{Q}_{12} \\ \mathbf{Q}_{12}{}^T & \mathbf{Q}_{22} \end{bmatrix}$ is positive, where $\mathbf{Q}_{11}$, $\mathbf{Q}_{12}$, $\mathbf{Q}_{22}$ are obtained from (9)-(11).

Proof Denote

$$\alpha(t) = e^{\mathbf{A}t}, \ \beta(t) = \int_0^t e^{\mathbf{A}\tau} \mathbf{B} d\tau .$$

Then for $\forall \mathbf{x} \in \mathbf{R}^n, \mathbf{y} \in \mathbf{R}^m$,

$$\begin{bmatrix} \mathbf{x}^T & \mathbf{y}^T \end{bmatrix} \begin{bmatrix} \mathbf{Q}_{11} & \mathbf{Q}_{12} \\ \mathbf{Q}_{12}{}^T & \mathbf{Q}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \mathbf{y} \end{bmatrix}$$
$$= \mathbf{x}^T \mathbf{Q}_{11} \mathbf{x} + \mathbf{y}^T \mathbf{Q}_{12}{}^T \mathbf{x} + \mathbf{x}^T \mathbf{Q}_{12} \mathbf{y} + \mathbf{y}^T \mathbf{Q}_{22} \mathbf{y}$$
$$= \int_0^h \mathbf{x}^T \alpha^T(t) \mathbf{Q} \alpha(t) \mathbf{x} dt + \int_0^h \mathbf{y}^T \beta^T(t) \mathbf{Q} \beta(t) \mathbf{y} dt$$
$$+ \int_0^h \mathbf{x}^T \alpha^T(t) \mathbf{Q} \beta(t) \mathbf{y} dt + \int_0^h \mathbf{y}^T \beta^T(t) \mathbf{Q} \alpha(t) \mathbf{x} dt$$
$$+ \mathbf{y}^T (h\mathbf{R}) \mathbf{y}$$

$$= \|\alpha(t)\mathbf{x}\|_\mathbf{Q}^2 + \|\beta(t)\mathbf{y}\|_\mathbf{Q}^2 + \langle \alpha(t)\mathbf{x}, \beta(t)\mathbf{y} \rangle + \mathbf{y}^T (h\mathbf{R}) \mathbf{y}$$
$$= \|\alpha(t)\mathbf{x} + \beta(t)\mathbf{y}\|_\mathbf{Q}^2 + \mathbf{y}^T (h\mathbf{R}) \mathbf{y} \tag{16}$$

Note that $\mathbf{Q}>0$ and $\mathbf{R}>0$, hence, $\begin{bmatrix} \mathbf{Q}_{11} & \mathbf{Q}_{12} \\ \mathbf{Q}_{12}{}^T & \mathbf{Q}_{22} \end{bmatrix}$ is positive-definite. □

Theorem 2 For stable $\mathbf{A}_c$ and $M \geq 1$, assume the sampling period h is not pathological, then the model predictive controller given by (5)-(8) is stabilizing.

Proof Since $\mathbf{A}_c$ is stable and h is not pathological, ($\mathbf{A}_d$, $\mathbf{B}_d$) is stable too.
Let $\mathbf{x}^*[k+j|k]$, $\mathbf{u}^*[k+j|k]$ denote the optimal state and input at time $(k+j)h$ computed from (5-8) at time kh. The optimal value of the objective function at time kh is then

ISBN: 0 08 043248 4

$$J^*[k] = \mathbf{x}^{*T}[k]\mathbf{Q}_{11}\mathbf{x}^*[k] + 2\mathbf{x}^{*T}[k]\mathbf{Q}_{12}\mathbf{u}^*[k] + \mathbf{u}^{*T}[k]\mathbf{Q}_{22}\mathbf{u}^*[k] + \int_{(k+1)h}^{+\infty}\left[\mathbf{x}^{*T}(t)\mathbf{Q}\mathbf{x}^*(t) + \mathbf{u}^{*T}(t)\mathbf{R}\mathbf{u}^*(t)\right]dt \tag{17}$$

Suppose there is no disturbances, then at time $(k+1)h$, the initial condition of the state is

$$\mathbf{x}[k+1] = \mathbf{x}^*[k+1|k].$$

Therefore the objective function is

$$J[k+1] = \int_{(k+1)h}^{+\infty}\left[\mathbf{x}^{*T}(t)\mathbf{Q}\mathbf{x}^*(t) + \mathbf{u}^{*T}(t)\mathbf{R}\mathbf{u}^*(t)\right]dt$$

if the following input sequence is used at time $(k+1)h$

$$\{\mathbf{u}^*[k+1|k], \mathbf{u}^*[k+2|k], \cdots, \mathbf{u}^*[k+M-1|k], \mathbf{0}\}.$$

Since the optimal objective function $J^*[k+1]$ at time $(k+1)h$ is no worse than $J[k+1]$, we have

$$J^*[k+1] \le J[k+1] = J^*[k] - J_0 = J^*[k] - \begin{bmatrix}\mathbf{x}^T[k] & \mathbf{u}^T[k]\end{bmatrix}\begin{bmatrix}\mathbf{Q}_{11} & \mathbf{Q}_{12}\\ \mathbf{Q}_{12}^T & \mathbf{Q}_{22}\end{bmatrix}\begin{bmatrix}\mathbf{x}[k]\\ \mathbf{u}[k]\end{bmatrix} \tag{18}$$

The sequence $J^*[k]$ is therefore nonincreasing. It converges because it bounded below by zero. From Lemma 2, we have that both $x[k]$ and $u[k]$ converge to zero for $k > k_0$, which is large enough. So the controller is stabilizing from (4). □

Remark 4. We only give the algorithm and the stability proof for stable systems. However, it is easy to extend these results to unstable systems in the same way as Rawlings (1993). In this case, the problem can be solved via dynamic programming with constraints.

Remark 5. The phenomena of a pathological sample period can be found in Chen and Francis (1996).

5 CONCLUSION

The model predictive controller for sampled-data systems is presented in this paper. The most attractive advantage of the proposed controller is that it considers the intersample behavior of the system and shows a better performance than that in the discrete-time case. The controller is also stabilizing as the same as in the discrete-time case.

REFERENCE

Bamieh, B. and J. B. Pearson (1992a). A General Framework for Linear Periodic Systems with Applications to H_∞ Sampled-Data Control. *IEEE Trans. Automat. Contr.*, AC-**37**, 418-435.

Bamieh, B. and J. B. Pearson (1992b). The H_2 Problem for Sampled-Data Systems. *Syst. Control Lett.*, **19**, 1-12.

Camacho, E. F. and C. Bordons (1995). *Model Predictive Control in the Process Industry.* Lecture Notes in Control and Information Sciences. Springer-Verlag, New York.

Chen T. and B. A. Francis (1991). H_2-optimal Sampled-Data Control. *IEEE Trans. Automat. Contr.*, AC-**36**, 387-397.

Chen T. and B. A. Francis (1995). *Optimal Sampled-Data Control Systems*, Springer, 1995.

Chen T. and B. A. Francis (1996). H_∞-Optimal Sampled-Data Control: Computation and Design. *Automatica*, **32**, 223-228.

Clarke, D. W. and R. Scattolini (1991). Constrained Receding-Horizon Predictive Control. *IEE Proceedings Part D*, **138**, 347-354.

Dullerud, G. and K. Glover (1993). Robust Stabilizaion of Sampled-Data Systems to Structured LTI Perturbations. *IEEE Trans. Automat. Contr.*, AC-**38**, 1497-1508.

Dullerud, G. and K. Glover (1996). Robust Performance of Periodic Systems. *IEEE Trans. Automat. Contr.*, AC-**41**, 1146-1159.

Garcia, C. E., D. M. Prett and M. Morari (1989). Model Predictive Control - A Survey. *Automatica*, **25**, 335-348.

Genceli, H. and M. Nikolaou (1993). Robust Stability Analysis of Constrained l_1-norm Model Predictive Control. *AIChE J.* **39** 1954-1965.

Hagiwara, T. and M. Araki (1995). FR-operator Approach to the H_2 Analysis and Synthesis. *IEEE Trans. Automat. Contr.*, AC-**40**, 1411-1421.

Hara, S., Y. Yamamoto and H. Fujioka (1996). Modern and Classical Analysis / Synthesis Methods in Sampled-Data Control 桝 Brief Overview with Numerical Examples. *Proc. 36th IEEE CDC*, 1251-1256.

Hayakawa, Y., S. Hara and Y. Yamamoto (1994). H_∞ Type Problem for Sampled-Data Control Systems - A Solution via Minimum Energy Characterization. *IEEE Trans. Automat. Contr.*, AC-**39**, 2278-2284.

Kabamba, P. T. and S. Hara (1993). Worst Case Analysis and Design of Sampled Data Control Systems. *IEEE Trans. Automat. Contr.*, AC-**38**, 1337-1357.

Khammash, M. H. (1993). Necessary and Sufficient Conditions for the Robustness of Time-Varying Systems with Applications to Sampled-Data Systems. *IEEE Trans. Automat. Contr.*, AC-**38**, 49-57.

Khargonekar, P. P and N. Sivashankar (1991). H_2 Optimal Control for Sampled-Data Systems. *Syst. Control Lett.*, **17**, 425-436.

Kothare, M. V., V. Balakrishnan and M. Morari

ISBN: 0 08 043248 4

(1996). Robust Constrained Model Predictive Control using Linear Matrix Inequalities. *Automatica*, **32**, 1361-1379.

Kouvaritakis, B., J. A. Rossiter, A. O. T. Chang (1992). Stable Generalised Predictive Control: an algorithm with guaranteed stability. *IEE Proceedings Part D*, **139**, 349-362.

Kwon, W. H. (1994). Advances in Predictive Control: Theory and Application. *'94 Asian Control Conference*.

Rawlings, J. B., and K. B. Muske (1993). The Stability of Constrained Receding Horizon Control. *IEEE Trans. Automat. Contr.* , AC-**38**, 1512-1516.

Richalet, J. (1993). Industrial Applications of Model Based Predictive Control. *Automatica*, **29**, 1251-1274.

Sivashankar, N. and P. P. Khargonekar (1993). Robust Stability and Performance Analysis of Sampled-Data Systems. *IEEE Trans. Automat. Contr.* , AC-**38**, 58-69.

Sivashankar, N. and P. P. Khargonekar (1994). Characterization and Computation of the L_2-induced Norm of Sampled-Data Systems. *SIAM J. Contr. and Optimiz.*, **32**, 1128-1150.

Sun, W. and K. M. Nagpal and P. P. Khargonekar (1993). H_∞ Control and Filtering for Sampled-data Systems. *IEEE Trans. Automat. Contr.* , AC-**38**, 1162-1175.

Toivonen, H. T. (1992). Sampled-Data Control of Continuous-time Systems with an H_∞ Optimality Criterion. *Automatica*, **28**, 45-54.

Zafiriou, E. (1990). Robust Model Predictive Control of Processes with Hard Constraints. *Comp. Chem. Eng.*, **14**, 359-371.

Zheng, A. and M. Morari (1995). Stability of Model Predictive Control with Mixed Constraints. *IEEE Trans. Automat. Contr.* , AC-**40**, 1818-1823.

Zheng, Z. Q. and M. Morari (1993). Robust Stability of Constrained Model Predictive Control. In *Proc. American Control Conf.*, San Francisco, CA, 379-383.

ISBN: 0 08 043248 4