A Majorization Condition for MIMO Stabilizability via MIMO Transceivers with Pure Fading Subchannels

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Abstract: This paper aims at characterizing a fundamental limitation on the information constraints required for multi-input networked stabilization. A MIMO communication system is deployed for information exchange between the controller and the plant. The communication system is modeled as a MIMO transceiver, which consists of three parts: an encoder, a MIMO channel consisting of parallel SISO subchannels, and a decoder. We focus on the pure fading subchannels in this paper while the case of pure AWGN subchannels has been discussed in our previous work. Inheriting the spirit of MIMO communication, the number of SISO subchannels in the transceiver is often greater than the number of control inputs to be transmitted. The subchannel capacities are assumed to be fixed a priori. With the encoder/decoder pair at hand, the controller designer gains an additional design freedom on top of the controller, leading to a stabilization problem via coding/control co-design. A necessary and sufficient condition is obtained for the solvability of this coding/control co-design problem given in terms of a majorization type relation. A numerical example is presented to illustrate our results.

Key Words: Networked stabilization, MIMO communication, coding/control co-design, majorization, topological entropy.

1 Introduction

A networked control system (NCS) is a feedback system wherein the feedback loop is closed over a communication network subject to various information constraints such as data rate constraint [1, 17], quantization [7, 15], fading [5], and communication delay [10], etc. One primary issue in networked control is to characterize a fundamental limitation on the information constraints required for networked stabilization. Such information requirement has now been rather understood for single-input systems after numerous investigations under different information constraints. See [1, 17, 20] for data rate constraint, [6, 7] for quantization, [5] for fading, [2] for signal-to-noise ratio constraint, etc. All these studies converge to a unified fundamental limitation on the information constraints required for stabilization given in terms of the topological entropy of the open-loop plant, i.e., the logarithm of the absolute product of unstable poles for a discrete-time plant, or the sum of the unstable poles for a continuous-time plant.

As for the information requirement for multi-input networked stabilization, consensus is far from reached in the research community while many nice attempts have been reported in the literature recently. See for instance [3, 7, 9, 18, 19, 22, 24]. In particular, the idea of channel/controller co-design was initiated in [18] and followed by several other works, for instance, [3, 24], to address the multi-input networked stabilization. The essence therein is to employ the twist of channel resource allocation, i.e., assuming that the channel capacities can be allocated among different input channels subject to a total capacity constraint. By virtue of this additional design freedom, the minimum total channel capacity required for networked stabilization is shown to be given by the topological entropy of the open-loop plant. A similar idea, although not stated explicitly, can be seen in [19] which considers networked stabilization over parallel Gaussian channels subject to a total power constraint.

While most of the existing studies assume a SISO communication scheme between the controller and the plant, the huge success of MIMO technology in communication theory and practice has caught our attention. Generally speaking, a MIMO communication system refers to a multi-input multi-output communication structure deployed to break the capacity limit of the conventional SISO communication scheme. It has been widely used in wireless communication where spacial diversity can be exploited to increase the data transmission capacity [21]. Our of curiosity, we raise the following questions: What will happen if MIMO control meets MIMO communication? Will that lead to more design flexibilities and bring in new advantages?

Driven by these questions, we have initiated the use of MIMO communication systems for information exchange in networked control in our previous work [4]. Therein the networked stabilizability via MIMO transceivers with pure AWGN subchannels has been investigated. In this paper, we shall extend the study to the case of pure fading subchannels. To be more specific, the control inputs are now transmitted through a MIMO transceiver which consists of an encoder, a MIMO channel, and a decoder. One essence of MIMO communication is that the number of SISO subchannels in the transceiver is often greater than the number of data streams. When applied to networked control, this means that the number of subchannels is greater than the number of control inputs. We assume that the subchannel capacities are fixed a priori and, thus, cannot be freely allocated as in [3, 18, 24]. Nevertheless, the encoder/decoder pair now gives a substituted design freedom. The controller designer needs to design the encoder/decoder pair and the controller jointly so as to stabilize the system, leading to a stabilization problem via coding/control co-design.

To one's delight, the utilization of MIMO communication does bring in new vitality to the study of networked control. A fundamental limitation has been obtained in [4] for the

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information requirement for networked stabilizability via MIMO communication over AWGN subchannels given in terms of a majorization type condition. In this paper, we show that the same fundamental limitation also holds in the case of pure fading subchannels. Notice that majorization is a rather old topic in mathematics [11] and has been frequently used in statistics in the past 100 years. However, its engineering applications only appear recently, notably in wireless communication, information theory, operations research, and smart grid, etc. The application of majorization in control theory remains quite scattered in the literature. One relevant work can be seen in [14], in which majorization is utilized to investigate the remote state estimation with communication costs for a first-order linear time-invariant system.

The rest of this paper is organized as follows. Section 2 formulates the problem to be studied. Section 3 gives some preliminary knowledge. The main result of this work is presented in Section 4. A numerical example is worked out in Section 5. Finally, the paper is concluded in Section 6 with some perspectives. Most notation in this paper is more or less standard and will be made clear as we proceed.

2 Problem Formulation

Consider the NCS shown in Fig. 1. Here, the plant $[A|B]$ is a continuous-time linear time invariant system described by the state space model

$$\dot{x}(t) = Ax(t) + Bu(t), \quad x(0) = x_0,$$

where $A \in \mathbb{R}^{n\times n}$ and $B \in \mathbb{R}^{n\times m}$. Assume that $[A|B]$ is unstable but stabilizable. Let the state $x(t)$ be available for feedback. Due to the transmission error in the communication network, the received control signal $u(t)$ is only a distorted version of $v(t)$. In this case, whether feedback stabilization can be achieved critically depends on the transmission accuracy of the communication network. In fact, a general concern of networked stabilization is to find a fundamental limitation on the quality of the communication network so as to render stabilization possible.

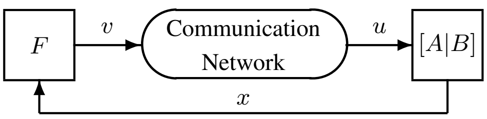

Figure 1: State feedback via communication network.

Note that most existing studies on multi-input networked stabilization [3, 7, 9, 18, 19, 24] assume a SISO communication scheme between the controller and the plant, i.e., each control input is transmitted via a dedicated SISO channel. On the contrary, in this paper, we are motivated by the huge success of MIMO communication and wish to explore the potential advantages brought about by utilizing a MIMO communication system in networked control.

A typical MIMO communication system, also referred to as a MIMO transceiver, is depicted in Fig. 2. It consists of three parts: an encoder matrix T, a MIMO channel, and a decoder matrix R, where the MIMO channel is characterized by a channel matrix H and a multiplicative stochastic noise κ followed by an additive white Gaussian noise d. The communication engineers aim at designing the encoder and the decoder so as to make the received signal u approximate the transmitted signal v as accurately as possible. To make the most of the advantages of MIMO communication, the transceiver is often built in such a way that the dimensions of q and p are much higher than the dimension of v and u.

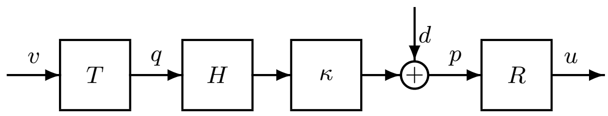

Figure 2: MIMO transceiver, a typical MIMO communication system.

In connection with the NCS concerned, we are curious to ask: What will happen if MIMO communication is used in networked control? Does it offer new advantages? Does it lead to new design flexibilities?

Another main motivation of this work comes from the following concern. Recall that the channel resource allocation as in [3, 18, 24] is based on a crucial assumption that the channel capacities can be allocated among different input channels subject to a total capacity constraint. What if the individual channel capacities are indeed given a priori and not allocatable? In that case, is it possible to explore some other design freedoms so as to stabilize the NCSs?

Both motivations lead us to the problem below. Instead of using a SISO communication scheme, we use a MIMO transceiver as shown in Fig. 2 to transmit the control signals. For simplicity, we assume that the channel matrix H is identity. In fact, all the later developments can be extended directly to the case of a known nonsingular H. When H is identity, the MIMO channel simply becomes a collection of l parallel general fading SISO subchannels, where each subchannel is characterized by a multiplicative stochastic noise κ_i cascaded with an additive noise d_i. In the spirit of MIMO communication, we assume that the number of SISO subchannels is greater than or equal to the number of data streams to be transmitted, i.e., $l \geq m$. Later we will see that $l < m$ may also be valid in some cases. The current communication system is shown in Fig. 3.

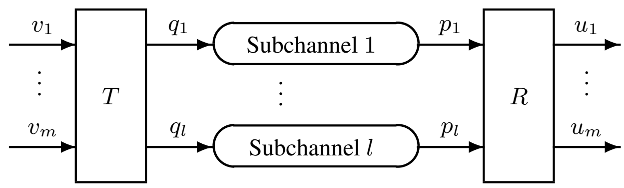

Figure 3: A MIMO transceiver as a MIMO communication system in MIMO control.

We focus on the pure fading effect in the subchannels in this paper. The case of pure AWGN subchannels has been discussed in a parallel work [4] wherein a necessary and sufficient condition has been obtained for the networked stabilizability. Later in this paper we indicate that the same condition holds in the case of pure fading subchannels except for a different definition of subchannel capacities. The case of general fading subchannels with both multiplicative noise and additive noise is under our current investigation.

Specifically, now consider each SISO subchannel as a pure fading channel as in Fig. 4, where the input signal

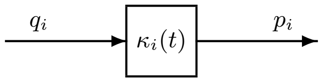

Figure 4: A pure fading subchannel.

q_i is subject to a stochastic multiplicative white noise $\kappa_i(t)$ with mean $\boldsymbol{E}[\kappa_i(t)] = \mu_i > 0$ and autocovariance $\boldsymbol{E}[(\kappa_i(t) - \mu_i)(\kappa_i(t+\tau) - \mu_i)] = \sigma_i^2 \delta(\tau)$. The mean-square capacity of such a fading channel is defined as

$$\mathfrak{C}_i = \frac{1}{2}\frac{\mu_i^2}{\sigma_i^2}. \tag{1}$$

The total channel capacity is given by $\mathfrak{C} = \mathfrak{C}_1 + \mathfrak{C}_2 + \cdots + \mathfrak{C}_l$. We assume that the subchannel capacities are fixed a priori. However, we do not assume any kind of monotonicity among them. For future use, denote

$$M = \mathrm{diag}\{\mu_1, \mu_2, \ldots, \mu_l\}, \quad \Sigma^2 = \mathrm{diag}\{\sigma_1^2, \sigma_2^2, \ldots, \sigma_l^2\}.$$

Since the subchannel capacities are now fixed a priori, the channel resource allocation as in [3, 18, 24] can no longer be performed. Nevertheless, the encoder matrix T and decoder matrix R in the MIMO transceiver now serve as substituted design freedoms. They can be freely designed subject to a mild constraint:

$$RMT = I,$$

which simply says that the received signal u has the same mean as the transmitted signal v. The controller designer is to jointly design the controller and the encoder/decoder pair so as to stabilize the system, leading to a stabilization problem via coding/control co-design.

With the MIMO transceiver in the loop, the closed-loop system takes the form as shown in Fig. 5, where $\kappa(t) = \mathrm{diag}\{\kappa_1(t), \kappa_2(t), \ldots, \kappa_l(t)\}$ stands for the stochastic multiplicative noises in the fading subchannels. The state-space

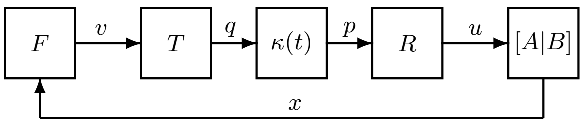

Figure 5: NCS with MIMO communication.

description of the closed-loop system is given by

$$\dot{x}(t) = [A + BR\kappa(t)TF]x(t),$$

which, to be rigorous, should be written in standard Itô form:

$$dx(t) = (A+BF)x(t)dt + \sum_{i=1}^{l} \sigma_i B R_i T_i F x(t) d\omega_i(t),$$

where R_i is the ith column of R, T_i is the ith row of T, and $\omega_i, i = 1, 2, \ldots, l$, are independent Wiener processes. Denote by $X(t) = \boldsymbol{E}[x(t)x(t)']$ the state covariance. By Itô's formula [13], the evolution of $X(t)$ is governed by the following matrix differential equation:

$$\dot{X}(t) = (A+BF)X(t) + X(t)(A+BF)' + BR[\Sigma^2 \odot (TFX(t)F'T')]R'B',$$

where $\odot$ means Hadamard product. We say that $[A|B]$ is mean-square stabilizable if there exist a state feedback gain F and an encoder/decoder pair such that the closed-loop system is mean-square stable, i.e., $\lim_{t\to\infty} X(t) = 0$.

We are interested in finding a fundamental limitation on the subchannel capacities $\mathfrak{C}_i, i = 1, 2, \ldots, l$, such that the networked stabilization can be accomplished.

Before proceeding, let us recall the notion of topological entropy [1–3, 19] of a continuous-time linear system $\dot{x}(t) = Ax(t)$, which is defined as the quantity $H(A) = \sum_{\Re(\lambda_i)>0} \lambda_i$, where λ_i are the eigenvalues of A.

3 Preliminary

Some preliminary knowledge is presented in this section for preparation.

3.1 Cyclic decomposition

Let A be an $n \times n$ real matrix. Its minimal polynomial is defined to be the monic polynomial $\alpha(\lambda)$ of least degree such that $\alpha(A) = 0$. The matrix A is said to be cyclic if its minimal polynomial has degree n. The following lemma gives the cyclic decomposition of a linear system, which plays an essential role in later developments.

Lemma 1 *Given a stabilizable linear system $[A|B]$ with $A \in \mathbb{R}^{n\times n}$ and $B \in \mathbb{R}^{n\times m}$, there exist nonsingular matrices P and Q such that*

$$[P^{-1}AP|P^{-1}BQ] = \left[\begin{bmatrix} A_1 & 0 & \cdots & 0 \\ 0 & A_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & A_k \end{bmatrix}\middle|\begin{bmatrix} b_1 & * & \cdots & * & * \\ 0 & b_2 & \ddots & \vdots & \vdots \\ \vdots & \ddots & \ddots & * & * \\ 0 & \cdots & 0 & b_k & * \end{bmatrix}\right], \tag{2}$$

where $A_i, i = 1, 2, \ldots, k$, are cyclic with minimal polynomials $\alpha_i(\lambda)$, such that $\alpha_1(\lambda) = \alpha(\lambda)$ and $\alpha_{i+1}(\lambda)|\alpha_i(\lambda)$ for $i = 1, 2, \ldots, k-1$. Moreover, the subsystems $[A_i|b_i], i = 1, 2, \ldots, k$, are stabilizable.

For the details, one can refer to [8] for the cyclic decomposition of a matrix and [23] for the cyclic decomposition of a linear system. Note that the number k as in Lemma 1 is referred to as the cyclic index of A and is unique. The minimal polynomials $\alpha_i(\lambda), i = 1, 2, \ldots, k$, are also unique. In addition, from the relation $\alpha_{i+1}(\lambda)|\alpha_i(\lambda)$, it follows that the spectrum of A_{i+1} is contained in the spectrum of A_i. Consequently, there naturally holds $H(A_1) \geq H(A_2) \geq \cdots \geq H(A_k)$.

Remark 1 *The subsystems $[A_i|b_i], i = 1, 2, \ldots, k$, are hereinafter referred to as the cyclic subsystems of the system $[A|B]$. The role of nonsingular matrices P and Q can be considered as linear transformations in the state space and input space, respectively. The following implication can be inferred from Lemma 1. In the cyclic decomposition (2), A_1 contains the greatest number of unstable eigenvalues of A that can be stabilized by a single control input up to linear transformations in the input space; likewise, A_1 together with A_2 contains the greatest number of unstable eigenvalues of A that can be stabilized by two control inputs* *up to linear transformations in the input space; and so on so forth.*

3.2 Majorization

For $x, y \in \mathbb{R}^n$, we denote by $x^\uparrow$ and $y^\uparrow$ the rearranged versions of x and y so that their elements are arranged in a non-decreasing order. We say that x is majorized by y, denoted by $x \preccurlyeq y$, if

$$\begin{cases} \sum_{i=1}^{k} x_i^\uparrow \geq \sum_{i=1}^{k} y_i^\uparrow, \quad k = 1, 2, \ldots, n-1, \\ \sum_{i=1}^{n} x_i^\uparrow = \sum_{i=1}^{n} y_i^\uparrow. \end{cases} \tag{3}$$

If the last equality in (3) is changed to an inequality $\geq$, then x is said to be weakly majorized by y from above, denoted by $x \preccurlyeq^w y$. Furthermore, if all the inequalities $\geq$ in (3), including the last equality, are changed to strict inequalities $>$, then x is said to be strictly weakly majorized by y from above, denoted by $x \prec^w y$. Note that when two vectors are compared via majorization or weak majorizations, the order of the elements in the vectors is irrelevant.

The physical interpretation of majorization is often very interesting in applications. It orders the level of fluctuations when the averages are the same. In other words, $x \preccurlyeq y$ says that the elements of x are more even or, less spread out, than the elements of y. For a comprehensive treatment of majorization, one can refer to [16].

4 Main Result

An explicit characterization of information requirement for multi-input networked stabilization via MIMO communication is presented in the following theorem. The proof is omitted here for brevity.

Theorem 1 *$[A|B]$ is mean-square stabilizable via MIMO communication, if and only if*

$$\begin{bmatrix}\mathfrak{C}_1 & \mathfrak{C}_2 & \cdots & \mathfrak{C}_l\end{bmatrix}' \prec^w \begin{bmatrix}H(A_1) & H(A_2) & \cdots & H(A_k) & 0 & \cdots & 0\end{bmatrix}', \tag{4}$$

where $H(A_i), i = 1, 2, \ldots, k$, are the topological entropies of the cyclic subsystems $[A_i|b_i]$ as in (2).

Two interesting corollaries are deduced from Theorem 1 as below. The proofs are omitted here for brevity.

Corrollary 1 *If the cyclic decomposition of A has only one unstable cyclic block, i.e., A_1, then $[A|B]$ is mean-square stabilizable via MIMO communication if and only if $\mathfrak{C} > H(A)$.*

Notice that Corollary 1 includes the single-input system as a special case. It simply suggests that in stabilizing a single-input system via MIMO communication, a total capacity requirement is all one needs. How the individual subchannel capacities are distributed is irrelevant in this case.

Corrollary 2 *If $\mathfrak{C}_1 = \mathfrak{C}_2 = \cdots = \mathfrak{C}_l$, then $[A|B]$ is mean-square stabilizable via MIMO communication if and only if $\mathfrak{C} > H(A)$.*

Corollary 2 somehow suggests that identical subchannels can best help each other in transmitting the signals.

Remark 2 *From the majorization type condition (4), one can observe that in some cases, it may also be possible to stabilize the NCS with less number of SISO subchannels than the number of control inputs. The minimum number of SISO subchannels needed for stabilization is equal to the number of unstable cyclic subsystems $[A_i|b_i]$ yielded from the cyclic decomposition (2). This observation is consistent with earlier studies [12, 23] in the literature that investigate the minimum number of control inputs required to stabilize a linear system. In that aspect, our result strengthens those studies by indicating a fundamental limitation on the information constraints required for networked stabilization given in terms of a majorization type relation.*

5 Example

A numerical example is given in this section to illustrate the process of coding/control co-design.

Consider the following unstable system $[A|B]$:

$$A = \begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \\ 0 & 1 \end{bmatrix},$$

with initial condition $x_0 = \begin{bmatrix}1 & 1 & 1 & 1\end{bmatrix}'$. Clearly, $[A|B]$ is stabilizable. Moreover, it is already in the cyclic decomposition form (2) with cyclic subsystems

$$[A_1|b_1] = \left[\begin{bmatrix} 4 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \middle| \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\right], \text{ and } [A_2|b_2] = [1|1].$$

It follows that $H(A_1) = 4 + 2 + 1 = 7$, and $H(A_2) = 1$.

Consider the case when the MIMO transceiver has three fading SISO subchannels specified by

$$M = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 0.6 & 0 \\ 0 & 0 & 0.9 \end{bmatrix}, \quad \Sigma^2 = \begin{bmatrix} 0.35 & 0 & 0 \\ 0 & 0.2 & 0 \\ 0 & 0 & 0.25 \end{bmatrix}.$$

In view of (1), the subchannel capacities are

$$\mathfrak{C}_1 = 5.7143, \ \mathfrak{C}_2 = 0.9, \text{ and } \mathfrak{C}_3 = 1.62.$$

One can easily verify that the strictly weak majorization relation (4) is satisfied and, thus, the mean-square stabilization can be accomplished via certain coding/control co-design. One such co-design is carried out as below.

For the controller design, we solve the $\mathcal{H}_2$ optimal complementary sensitivity for each of the two cyclic subsystems $[A_i|b_i], i = 1, 2$, i.e., solving the following optimal control problems:

$$\inf_{f_i : A_i + b_i f_i \text{ is stable}} \|\boldsymbol{T_i}(s)\|_2,$$

where $\boldsymbol{T}_i(s) = f_i(sI - A_i - b_i f_i)^{-1} b_i$. This yields two optimal feedback gains $f_1 = \begin{bmatrix}-40 & 36 & -10\end{bmatrix}$ and $f_2 = -2$, respectively. Let

$$F = \text{diag}\{f_1, f_2\} = \begin{bmatrix} -40 & 36 & -10 & 0 \\ 0 & 0 & 0 & -2 \end{bmatrix}.$$

As for the coding design, let

$$T = M^{-\frac{1}{2}}UD^{-1}, \text{ and } R = DU'M^{-\frac{1}{2}},$$

with

$$U = \begin{bmatrix} 0.8952 & -0.1993 \\ 0 & 0.8944 \\ 0.4456 & 0.4004 \end{bmatrix} \text{ and } D = \begin{bmatrix} 1 & 0 \\ 0 & 0.1 \end{bmatrix}.$$

Under this coding/control co-design, the closed-loop system is mean-square stable. As shown in Fig. 6, the Frobenius norm of the state covariance goes to zero asymptotically.

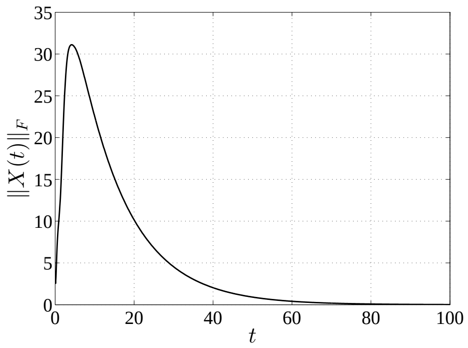

Figure 6: Closed-loop evolution of $\|X(t)\|_F$.

6 Conclusion

In this paper, we investigate the networked stabilization via MIMO communication, leading to a coding/control co-design problem. A fundamental limitation on the information requirement for multi-input networked stabilization is obtained which is given by a majorization type relation. We conclude this paper with some perspectives that echo the motivating questions raised in the very beginning. The use of MIMO communication in networked control does bring in new advantages and flexibilities:

1) The condition on the subchannel capacities for stabilizability is weakened to a great extent compared to the existing understanding. The condition is now given in terms of a strictly weak majorization relation as in (4).
2) The redundancy in the number of SISO subchannels helps reduce the capacity requirement in the individual subchannels. For instance, with MIMO communication, one can use a collection of subchannels with small capacities to stabilize a single-input system as long as the total capacity is greater than the topological entropy.
3) By virtue of the coding mechanism, in some cases, it may even be possible to stabilize the NCS with less number of subchannels than the number of the control inputs. The minimum number of subchannels needed is equal to the number of unstable cyclic subsystems yielded from the cyclic decomposition.

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