Differentiated Energy Services: Multiple Arrival Times and Multiple Deadlines ⋆

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Abstract: The supply/demand balance problem plays a pivotal role in the electricity grid, especially when an increasing proportion of power is generated from renewable resources. Enormous supply/demand models have been put forward in order to handle the balance problem in the smart grid, in the presence of high renewable penetration. This has brought an awareness that the flexibilities in the demand can be exploited to alleviate the burden on the supply. In view of this, we apply and study differentiated energy services, which distinguish demands in terms of their available flexibilities. As a starting point, we concentrate on two problems regarding adequacy. On the one hand, we find both numerical and analytical ways to check the adequacy of a supply. On the other hand, we characterize the adequacy gap in the case of an inadequate supply.

Keywords: Smart grids, supply/demand balance, demand response, $(0,1)$-matrix completion, flow network theory

1. INTRODUCTION

For the purpose of sustainable development (Azapagic and Perdan, 2011), more and more renewable resources, such as solar and wind energy, are being exploited to generate electricity. In spite of the alluring advantages of renewables, the inherent uncertainty and intermittency of renewable energy have inevitably posed great challenges to the establishment and maintenance of a sustainable power system. Particularly, this raises worldwide concerns about how to balance the supply and demand in consideration of the deeper penetration of renewables.

A natural approach is to compensate for the fluctuation in the demand by way of reserve generations. Such *supply side* approach has already been put into practice and also proven successful when the majority of power is still generated from traditional resources such as fossil fuels. However, due to the increasing amount of renewable generations, the supply side approach requires considerable quantities of reserves at the expense of both economical and environmental benefits. See, for instance, Helman et al. (2010), Ortega-Vazquez and Kirschen (2010), and Halamay et al. (2011).

With the growing development of the smart grid, the *demand side* approach, widely known as *demand response*, has raised the growing interest of engineers and scientists. It focuses on exploiting the flexibilities in demand to compensate for the undesirable attributes of renewable energy. Researchers are also more aware of the various flexibilities residing in different loads. Following are some typical examples of flexible loads: electrical vehicles, thermostatically controlled loads, residential pool pumps, commercial HVAC (heating, ventilation and air conditioning) systems and other smart appliances. Successful attempts at such loads have been made in Tan and Varaiya (1993), Clement-Nyns et al. (2010), Galus et al. (2010), Meyn et al. (2013), and Hao and Chen (2014), to name just a few. Loads may be deferable, intermittent or modulated depending on their respective natures and such flexibilities can make room for the volatilities of renewables. The GRIP (grids with intelligent periphery), proposed in Bakken et al. (2011), also provides a nice framework to carry out innovations regarding the demand response.

Along the line of demand approach, a number of creative supply/demand models have been proposed. Among them, we are particularly interested in the differentiated energy services, as described by Nayyar et al. (2016) and Chen et al. (2015). Generally speaking, the electricity is no longer treated as a homogenous product with a single unit price, but a set of energy services differentiated by levels of flexibility. As the name indicates, the duration-differentiated energy services in Nayyar et al. (2016) are differentiated by their durations only, while the duration-deadline jointly differentiated energy services in Chen et al. (2015) are jointly differentiated by both the duration requirements and deadlines. In both cases, the loads are assumed to be indifferent to the actual delivery time. Moreover, the power delivery rate is assumed to be a certain constant, rather than an interval of continuous real numbers. This is reasonable especially for some smart loads in smart grid, as shown in Yilmaz and Krein (2013). In such services, a day-ahead market is considered. Based

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on estimated supply and accumulated requirements from loads, the nominal provider schedules power delivery and makes deals with other electricity providers in advance.

Two significant issues have been discussed in both aforementioned works on differentiated energy services. One is the adequacy of supply, and the other is the market implementation. In this paper, we focus on the first issue with a further complicated yet more practical setup: the differentiated energy services with multiple arrival times and multiple deadlines. To start with, we establish the necessary and sufficient conditions under which the supply can satisfy the requirements from a set of loads. Finding such an adequacy condition is equivalent to characterizing the existence of a constrained $(0, 1)$-matrix, which is closely connected with a flow network. We show that the polynomial algorithms for maximal flow can be applied to check the adequacy of the supply and simultaneously generate a feasible allocation when the supply is indeed adequate. In addition, a closed-form condition is given in terms of the nonnegativity of a structure tensor. The analytical condition physically implies that the demand tails should always be dominated by the supply tails. This coincides with the common intuition that the demand should be dominated by adequate supply. In case the supply is inadequate, the adequacy gap will be derived from the difference between the total demand and the value of the maximal flow, or equivalently, the absolute value of the minimum element of the structure tensor. A simple algorithm is presented to find a minimal feasible purchase which makes the total supply adequate.

The rest of the paper is organized as follows. In Section 2, we introduce the supply/demand model studied in this paper and formulate the main problems. In Section 3, the main results are presented, with reference to both the adequacy condition and the adequacy gap. Finally, we conclude the paper and present some promising future extensions in Section 4. The notation used in this paper is mostly standard and will be made clear as we proceed. We use $\mathbb{O}$ to denote a matrix with all elements equal to zero, and $\mathbb{E}$ to denote a matrix with all elements equal to one. Given a number a, we denote $a^+ = \max\{a, 0\}$. For an assertion $\mathcal{A}$, $\mathbb{1}(\mathcal{A})$ is assigned the value 1 if $\mathcal{A}$ is true, and 0 otherwise.

2. PROBLEM FORMULATION

In this paper, we lay emphasis on two issues in differentiated energy services with multiple arrival times and deadlines. One is under what condition the energy supply is qualified to serve all the demands (adequacy condition), while the other is the minimum amount of additional purchase in case of an insufficient supply (adequacy gap). To resolve these issues, we firstly demonstrate the supply/demand model we rely on in this paper.

2.1 *Differentiated Energy Services: Multiple Arrival Times and Multiple Deadlines*

The operational horizon is divided into T consecutive time slots. At each time slot t, the available supply is denoted by h_t. In addition to the two special time instances 0 and

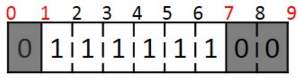

Fig. 1. An illustration of a service time specified by both the arrival time and the deadline

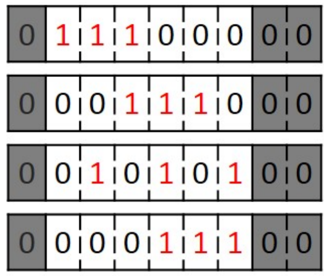

Fig. 2. Four qualified delivery results

T at both ends, the service provider points out $(\tau + 1)$ specified time instances, namely,

$$(0 = T_0) < T_1 < T_2 < \cdots < T_{\tau-1} < (T = T_\tau).$$

The demand arises from N consumers/loads, indexed by $i = 1, 2, \ldots, N$. The delivery rate from the supply to a load is constant, i.e., c units per time slot. Without loss of generality, let $c = 1$, which simply means the supply can allocate one or zero unit of power to a load at a time slot. Load i is characterized by a duration r_i and a service period specified by (a_i, d_i). This means that load i requires to be delivered r_i units within the time interval from the start of the $(T_{a_i} + 1)$th time slot to the end of the T_{d_i}th time slot. In other words, the supply has to deliver 1 unit of power to load i for r_i time slots within the respective service time. As shown in Fig. 1, over the $T = 9$ time slots in total, the specified time instances are indexed by 0, 1, 7, and 9, while the load cannot be served at the first, eighth or ninth time slots. Note that the load is indifferent to the actual delivery time provided the duration and service time requirements are satisfied.

We give four qualified power delivery results in Fig. 2, for a load i with $r_i = 3$ and the service time specified by Fig. 1. They are four different forms of the same service, among many other possible forms. Now, we can readily figure out why such services are called the differentiated energy services with multiple arrival times and deadlines, as each service is differentiated by the duration, the arrival time, and the deadline. When all the service times are presented by $(0, d_i)$, the case reduces to the duration-deadline jointly differentiated energy services in Chen et al. (2015). If further, all the loads have the same service time, specified by $(0, \tau)$, then the case reduces to the duration-differentiated energy services in Nayyar et al. (2016).

Remark 1. In the context, we assume that a series of time instances are specified by the nominal service provider in advance and consumers can only choose the service time determined by two of them. This is an operator-friendly scheme. Nevertheless, there is an alternative scheme, which seems more attractive to consumers. In this scheme, every consumer can choose a service time specified by any two time slots among the total T time slots. Then, those chosen as the arrival time or deadline are regarded as the specified time instances mentioned above. We claim that both schemes are equivalent up to our assumptions and results in this paper, though they have their respective strengths and weaknesses in other applications.

2.2 Mathematical Expressions for the Adequacy Condition and Adequacy Gap

Denote the *supply profile* and the *demand profile* by

$$\boldsymbol{h} = [h_1 \; h_2 \; \cdots \; h_T]', \quad \boldsymbol{r} = [r_1 \; r_2 \; \cdots \; r_N]',$$

respectively. Thus, the total supply and total demand are respectively denoted by

$$\sum_{t=1}^{T} h_t \text{ and } \sum_{i=1}^{N} r_i.$$

A supply profile $\boldsymbol{h}$ is adequate for a demand profile $\boldsymbol{r}$ under required service times if there exists a power allocation such that all the load requirements are satisfied. If further, the supply has no surplus after the appropriate allocation, then $\boldsymbol{h}$ is exactly adequate. We use an $N \times T$ pattern matrix F defined as follows to represent the constraints due to the different arrival times and deadlines. A position (i, t) is admissible if $T_{a_i} + 1 \le t \le T_{d_i}$. Otherwise, it is a forbidden position. The pattern matrix has 1's on all the admissible positions and 0's on all the forbidden positions. Physically, these admissible positions of the ith row denote when the power can be delivered to load i. It is easily verifiable that finding the adequacy condition is equivalent to characterizing the existence of a $(0, 1)$-matrix A satisfying the following constraints:

$$\sum_{i=1}^{N} A(i,t) \le h_t, \; \forall t = 1, 2, \ldots, T; \tag{1}$$

$$\sum_{t=1}^{T} A(i,t) = r_i, \; \forall i = 1, 2, \ldots, N; \tag{2}$$

$$\mathbb{O} \le A \le F, \tag{3}$$

The column sum constraints (1) are associated with the restricted supplies; the row sum constraints (2) correspond to the duration requirements of the loads; the required service times are consistent with the pattern matrix F. The pattern matrix is of a staircase pattern when it is restricted to the duration-deadline jointly differentiated energy services. Moreover, when $F = \mathbb{E}$, it reduces to the case of the duration-differentiated energy services. We denote the matrix class consisting of all such matrices by $\mathscr{A}(\boldsymbol{h}, \boldsymbol{r}, F)$. The claim below directly follows from the one-to-one correspondence between a feasible allocation and a matrix in $\mathscr{A}(\boldsymbol{h}, \boldsymbol{r}, F)$.

Claim 2. The set $\mathscr{A}(\boldsymbol{h}, \boldsymbol{r}, F)$ is nonempty if and only if the supply profile $\boldsymbol{h}$ is adequate.

The following example aims at illustrating the relationship between the differentiated energy services and zero-one matrices. There are four different loads and their service times are shown in Fig. 3. A feasible power allocation is also given. The corresponding supply profile, the demand profile, the pattern matrix, and the equivalent matrix are given as follows:

$$\boldsymbol{h} = [1\;1\;1\;2\;2\;1\;2\;1\;1]',$$

$$\boldsymbol{r} = [1\;2\;5\;4]',$$

$$F = \begin{bmatrix} 0&0&1&1&1&1&1&0&0 \\ 0&1&1&1&1&0&0&0&0 \\ 1&1&1&1&1&1&1&0&0 \\ 0&1&1&1&1&1&1&1&1 \end{bmatrix}$$

$$A = \begin{bmatrix} 0&0&0&1&0&0&0&0&0 \\ 0&0&0&1&1&0&0&0&0 \\ 1&1&1&0&1&0&1&0&0 \\ 0&0&0&0&0&1&1&1&1 \end{bmatrix}$$

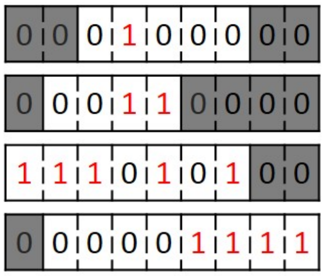

Fig. 3. An illustrative example

So far, we have reshaped the adequacy problem into an existence problem of a $(0, 1)$-matrix constrained by (1), (2), and (3). Researchers have paid close attention to the $(0, 1)$-matrix completion problem over the past century. See, for instance, Ryser (1957), Gale (1957), Mirsky (1971), Brualdi (1980), Anstee (1982), Anstee (1983), Brualdi and Dahl (2003), Marshall et al. (2011), and Chen et al. (2016). Apart from being a mathematical problem, it also has wide applications in engineering and social fields, for example, in the discrete tomography (Herman and Kuba, 2012), hard real-time computing systems (Buttazzo, 2011), and electoral systems (Lari et al., 2014). Quite a few elegant results have been obtained when the pattern matrix F is of certain special forms. Particularly, the Gale-Ryser theorem (Ryser 1957 and Gale 1957) solves the problem when $F = \mathbb{E}$, which is exactly the majorization condition for the duration-differentiated energy services in Nayyar et al. (2016). The results in Chen et al. (2016), where F is of a prescribed staircase pattern, lay a foundation for the study of the duration-deadline jointly differentiated energy services in Chen et al. (2015).

A follow-up question arises from an inadequate supply. A supplementary purchase profile $\boldsymbol{p}$ is feasible if it renders the total supply profile $\boldsymbol{p} + \boldsymbol{h}$ adequate. The adequacy gap g stands for the minimum supplementary purchase such that the total supply profile is adequate. Mathematically, finding the adequacy gap amounts to solving the following optimization problem:

$$\min_{\boldsymbol{p}} \; \sum_{t=1}^{T} p_t, \quad \text{subject to} \quad \boldsymbol{h} + \boldsymbol{p} \text{ being adequate,} \tag{4}$$

where $\boldsymbol{p} = [p_1 \; p_2 \; \cdots \; p_T]'$ is an integral vector, denoting the supplemental purchase over T time slots. The adequacy gap problem is one of the most fundamental problems that relate the differentiated energy services to the traditional electricity market. When the nominal provider in this paper faces a time-invariant unit purchasing price from the traditional market, the solution to the optimization problem (4) also minimizes the purchasing cost in order that the supply can provide qualified services for the given loads.

We assume that the information about the supply and demand are available offline. Therefore, the solvability of the above two problems will not be changed by permutating rows and/or columns of the pattern matrix F and modifying the supply and/or demand profiles correspondingly. Without loss of generality, the following assumption runs through this paper:

$$h_{T_i+1} \geq h_{T_i+2} \geq \cdots \geq h_{T_{i+1}}, \text{ for } i = 0, 1, \ldots, \tau - 1. \quad (5)$$

3. MAIN RESULTS

In this section, we firstly propose two necessary and sufficient conditions with respect to adequacy. One is given numerically, while the other is in analytic form. Then, based on the adequacy conditions, the adequacy gap problem is solved correspondingly. The ideas of proofs are sketched and the details can be found in the longer version of the paper available from the authors.

3.1 Adequacy Condition

In the last section, we transformed the adequacy problem into an existence problem of a $(0, 1)$-matrix constrained by (1), (2), and (3). Expanding on the existing works in the literature, we take one step forward to exploit the problem when the pattern matrix F is designated by the N loads with multiple arrival times and multiple deadlines. First of all, relax the constraint (2) to

$$\sum_{t=1}^{T} A(i, t) \leq r_i, \ \forall i = 1, 2, \ldots, N. \quad (6)$$

Then denote the matrix class with all the $(0, 1)$-matrices satisfying constraints (1), (3), and (6) by $\mathscr{A}_{\leq}(\boldsymbol{h}, \boldsymbol{r}, F)$. Before proceeding, we claim that we can find a matrix in $\mathscr{A}_{\leq}(\boldsymbol{h}, \boldsymbol{r}, F)$ with the maximal number (say $|\bar{f}|$) of 1's in polynomial time.

Proposition 3. The supply profile $\boldsymbol{h}$ is adequate if and only if $|\bar{f}| = \sum_{i=1}^{N} r_i$. If further, $\sum_{i=1}^{N} r_i = \sum_{t=1}^{T} h_t$, the supply is exactly adequate.

The key of the proof lies in the construction of an $s - t$ flow network, generated by the supply profile, the demand profile, and the pattern matrix. There exists a one-to-one correspondence between matrices in $\mathscr{A}_{\leq}(\boldsymbol{h}, \boldsymbol{r}, F)$ and feasible integral flows in the flow network. It can be shown that the well-known Ford-Fulkerson algorithm Ford and Fulkerson (1956) and/or the Edmonds-Karp algorithm Edmonds and Karp (1972) can help find a maximal integral flow in such a flow network. Let $|||F|||_1 = \sum_{i=1}^{N} \sum_{t=1}^{T} |F(i, t)|$. Then, the corresponding complexities of the above two algorithms under our notation are respectively given by

$$\mathcal{O}\left(|||F|||_1 |\bar{f}|\right) \text{ and } \mathcal{O}\left((N + T)|||F|||_1^2\right).$$

As a result, we can check the adequacy of the supply numerically. Moreover, if the supply is indeed adequate, it also generates a feasible allocation.

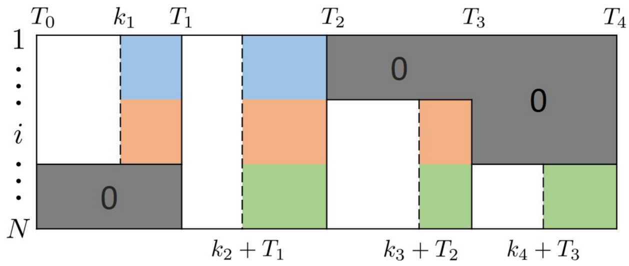

Fig. 4. Visual interpretation of a 4th-order structure tensor $W(\boldsymbol{h}, \boldsymbol{r}, F)$

Nevertheless, for the purpose of further analysis, like market implementation, an analytical necessary and sufficient condition is also desirable. Inheriting the merits of the method in Chen et al. (2016), we define an associated τth order tensor $W(\boldsymbol{h}, \boldsymbol{r}, F)$ of dimension $(T_1 + 1) \times (T_2 - T_1 + 1) \times \cdots \times (T_\tau - T_{\tau-1} + 1)$ as follows:

$$W_{k_1 k_2 \cdots k_\tau}(\boldsymbol{h}, \boldsymbol{r}, F) = \sum_{t>k_1}^{T_1} h_t + \sum_{t>T_1+k_2}^{T_2} h_t + \cdots + \sum_{t>T_{\tau-1}+k_\tau}^{T_\tau} h_t - \sum_{i=1}^{N} [r_i - (k_{a_i+1} + k_{a_i+2} + \cdots + k_{d_i})]^+, \quad (7)$$

where $0 \leq k_1 \leq T_1, 0 \leq k_2 \leq T_2 - T_1, \cdots, 0 \leq k_\tau \leq T - T_{\tau-1}$.

We call $W(\boldsymbol{h}, \boldsymbol{r}, F)$ a structure tensor, since it is merely determined by the structural information $(\boldsymbol{h}, \boldsymbol{r}, F)$. We say that $W(\boldsymbol{h}, \boldsymbol{r}, F)$ is nonnegative, if its every element is no less than zero, denoted by $W(\boldsymbol{h}, \boldsymbol{r}, F) \geq 0$.

Now, it is ready for us to elaborate our most significant result in this paper.

Theorem 4. The supply profile $\boldsymbol{h}$ is adequate if and only if $W(\boldsymbol{h}, \boldsymbol{r}, F) \geq 0$. If further, $\sum_{i=1}^{N} r_i = \sum_{t=1}^{T} h_t$, the supply is exactly adequate.

Alert readers may find that though the expression of the structure tensor may seem complicated, it indeed has regularity of its own. Hereinafter, we will unravel the underlying physical interpretations.

Fixing $k_1, k_2, \ldots, k_\tau$, we divide expression (7) into two parts by the minus sign. The first part is the summation of terms

$$\sum_{t>T_m+k_{m+1}}^{T_{m+1}} h_t,$$

for $m = 0, 1, \ldots, \tau - 1$. We call this the supply tail, whereas the second part is the summation of

$$[r_i - (k_{a_i+1} + k_{a_i+2} + \cdots + k_{d_i})]^+,$$

for $i = 1, 2, \ldots, N$, which is called the demand tail. Take Fig. 4 as an illustration, where $\tau = 4$. On the one hand, aggregating the colored part horizon, we obtain the supply tail. On the other hand, accumulating the colored part over the load index, we get the demand tail. As can be readily seen, the nonnegativity of $W(\boldsymbol{h}, \boldsymbol{r}, F)$ is nothing but the fact that the demand tail is dominated by the supply tail, for $0 \leq k_1 \leq T_1, 0 \leq k_2 \leq T_2 - T_1, \cdots$, and $0 \leq k_\tau \leq T - T_{\tau-1}$. Informally speaking, the energy dominance in tails implies the adequacy of the supply, and vice versa.

It appears that our adequacy condition stated in Theorem 4 is consistent with that shown in Nayyar et al. (2016) and Chen et al. (2015).

3.2 Adequacy Gap

So far, we are able to check whether a supply profile $\boldsymbol{h}$ is adequate or not. In the event of an inadequate supply, the next step is to find the adequacy gap.

Theorem 5. If the supply profile $\boldsymbol{h}$ is inadequate, then the adequacy gap g is given by

(1) $\sum_{i=1}^{N} r_i - |\bar{f}|$,
(2) or equivalently $\left|\min_{k_1,k_2,\cdots,k_\tau} W_{k_1 k_2 \cdots k_\tau}(\boldsymbol{h}, \boldsymbol{r}, F)\right|$.

Although the adequacy gap is determined once the structural information $(\boldsymbol{h}, \boldsymbol{r}, F)$ is given, the feasible supplementary purchase profile $\boldsymbol{p}$ which achieves this minimum may not be unique. For example, consider $\boldsymbol{h} = [1\ 1\ 1]'$, $\boldsymbol{r} = [2\ 2]'$, and $F = \mathbb{E}$. Apparently, the current supply profile is inadequate and there are three feasible supplementary purchase profiles, namely, $[0\ 0\ 1]'$, $[0\ 1\ 0]'$, and $[1\ 0\ 0]'$. The following algorithm can help us find a feasible supplementary purchase profile.

Algorithm 1. Find a minimal feasible purchase profile.

Input: the structural information triple $(\boldsymbol{h}, \boldsymbol{r}, F)$.

(1) Initialization: t=1; $\boldsymbol{p} = O$ of order $N \times 1$; $f_o^d = |\bar{f}| - \sum_{i=1}^{N} r_i$ or the minimal element of the structure tensor W in terms of $(\boldsymbol{p} + \boldsymbol{h}, \boldsymbol{r}, F)$. If $f_o^d = 0$, then output $\boldsymbol{p}$; otherwise, go to next step.
(2) $p_t = p_t + 1$; $f^d = |\bar{f}| - \sum_{i=1}^{N} r_i$ or the minimal element of the structure tensor W in terms of $(\boldsymbol{p} + \boldsymbol{h}, \boldsymbol{r}, F)$. Go to next step.
(3) Checker: If $f^d = 0$, then output $\boldsymbol{p}$. If $f^d \neq 0$ and $f^d > f_o^d$, then let $f_o^d = f^d$ and go to step (2); otherwise (i.e., $f^d = f_o^d \neq 0$), $p_t = p_t - 1$, $t = t + 1$, and return to step (2).

Output: a supplementary purchase profile $\boldsymbol{p}$.

Proposition 6. Algorithm 1 gives an optimal solution to the optimization problem (4).

Obviously, for the above example, the feasible supplementary purchase profile generated by Algorithm 1 is $[1\ 0\ 0]'$. In each iteration of this algorithm, the maximal flow in the updated flow network is augmented by at most one unit, so is the minimal element of the corresponding structure tensor. No redundant power will be purchased, under the supervision of the checker. If time-variant unit purchasing prices are given for each time slot, we leave for future work how to find a feasible supplementary purchase profile $\boldsymbol{p}$ to minimize the total purchasing cost.

4. CONCLUSION

On the basis of duration differentiated energy services and duration-deadline jointly differentiated energy services, we further study the differentiated energy services with multiple arrival times and multiple deadlines. Along this research line, we concentrate on two problems related to adequacy, namely, the adequacy condition and the adequacy gap problem. Relating the adequacy condition to a constrained $(0,1)$-matrix feasibility problem, we take a network flow approach to find the necessary and sufficient conditions for adequate supply profiles, both numerically and analytically. Essentially, the numerical method is to find a constrained allocation, which delivers as many units of power from the supply to the demand as possible. Thus, if the supply is indeed adequate, a feasible allocation is obtained as a by-product. The analytical condition is characterized by the nonnegativity of a structure tensor determined by the structural information triple $(\boldsymbol{h}, \boldsymbol{r}, F)$. The physical interpretation behind the expression (7) of a structure tensor is rather intuitive; i.e., the demand tail should always be dominated by the supply tail considering an adequate supply. To our delight, the adequacy gap can be correspondingly obtained by way of the adequacy conditions, as shown in Theorem 4. We also propose a simple algorithm to find one of the feasible supplementary purchase profiles.

In the future, we wish to explore the market implementation of differentiated energy services with multiple arrival times and multiple deadlines. We shall further find solutions to the minimum-cost purchase problem when unit prices are time-variant, as mentioned. There are other approaches in case of an insufficient supply. Under some circumstances, the nominal provider cannot purchase additional power from other grid markets. Consequently, it may reject making contracts with several customers in order that the rest demands can be accommodated well by the estimated supply.

Moreover, another possible direction is to introduce more load flexibilities into consideration. For instance, some preliminary work on the case with peer-to-peer charging allowed has been done in Mo et al. (2016). Also, numerical experiments imply that a causal allocation policy does not exist generally for the case with multiple arrival times and deadlines. In this regard, we shall make efforts to formulate suboptimal heuristic real-time scheduling policies for practical applications, where the supply profiles are only partly available.

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