Duration-differentiated Energy Services with Peer-to-peer Charging

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Abstract— **With the high penetration of renewable energy, the conventional solution to balancing supply and demand requires substantial reserve generations and, thus, curtails the environmental and financial benefits. To mitigate the overuse of reserves, an alternative approach, widely referred to as demand response, has been attracting increasing attention. The essence is to exploit the flexibility in demand to compensate for the variability in supply. Among the various forms of demand response, of particular interest to us is the so-called duration-differentiated energy service, in which a load requires a constant power level for a specified duration of time. In this paper, we further explore this by taking the charging/discharging interactions among the loads into account. The introduction of peer-to-peer charging facilitates power allocation and enlarges the set of adequate supply profiles. We propose an algorithm for power allocation and show that a given supply profile is adequate if and only if the algorithm produces a feasible allocation. We also relate this algorithm to dynamic programming. In the case of an inadequate supply, the adequacy gap can be obtained via a slightly modified algorithm.**

I. INTRODUCTION

In order to build a sustainable power system, the use of renewable resources is of central importance. The fact that the renewable energy production is inherently uncertain and intermittent makes the balance of demand and supply more challenging. How to achieve the demand/supply balance is believed to play a pivotal role in the increasing prevalence of renewable supply [1], [2].

The traditional way to balance supply and demand is a *supply side* approach, i.e., utilizing the reserve generation to compensate for the fluctuation in the demand. This strategy has proven to be successful when the majority of power is generated from traditional resources such as fossil fuels. However, as the proliferating renewables are integrated into the grid, the requirement for reserve generation is boosted significantly due to the uncertain and intermittent nature of renewables. Apart from being economically inefficient, the over-reliance on reserves may curtail the environmental benefits brought about by renewables since the reserve generation is mostly based on fossil fuels [3], [4].

While the conventional scheme is facing huge challenges, a burgeoning consensus suggests that the *demand side* approach is a promising alternative. The idea originates from the awareness of various flexibilities residing in different loads. To be specific, some loads can be deferred or intermitted, while some others can be modulated, depending on their respective natures. Typical examples include electric vehicles, thermostatically controlled loads, residential pool pumps, commercial HVAC systems and other smart appliances. The demand side approach, widely known as demand response, aims at exploiting the flexibility in the demand to compensate for the uncertainty in the supply. This idea has been reflected in the GRIP (Grids with Intelligent Periphery) architecture advocated in [5]. In this architecture, diverse geographically-dispersed resources are aggregated, while local scheduling and control strategies are implemented on the loads. Recently, many works have been reported, about modeling, characterizing, and utilizing the flexible loads. See, for instance, [6]–[10]. Moreover, a variety of market mechanisms have been designed to motivate consumers to elicit their flexibilities in return for financial compensation [6], [11]–[15]. Such markets provide differentiated energy services based on different levels of flexibility. This casts off the traditional market structure, which treats electricity as a homogenous product with a single unit price.

Following is a review of the existing literature closely related to the contents of this paper. Under the GRIP architecture, a duration-differentiated energy service was proposed in [14]. As indicated by name, the energy services are differentiated by their durations only. The loads are assumed to be indifferent to the actual delivery time, provided that their respective duration requirements are respected. This idea was extended in [15] and [16] to the duration-deadline jointly differentiated energy service, in which both duration and deadline are the factors that discriminate the energy services. There are two main issues discussed in this series of works. One aims at the problems related to adequacy. Analytical conditions have been established for a given supply profile to be adequate to satisfy all the load requirements. Meanwhile, algorithms for finding a feasible power allocation have been developed. The other issue is concerned with the market implementation of such differentiated energy services. It has been shown that there exists a forward market with an efficient competitive equilibrium with such services, as illustrated in [14] and [15].

Inspired by the works above, in this paper, we take one step forward and introduce the peer-to-peer charging to the duration-differentiated energy services. The motivation mainly comes from the rapid development of energy storage and transfer. These new developments in technology make the interactive charging among different loads not only technologically feasible but also economically practicable, as

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illustrated in [5] and [17]–[19]. The introduction of peer-to-peer charging enables the possibility of temporarily storing the power in some loads in service and then conveying it to different loads at a later time. We are driven to understand the potential advantages brought about by this additional dimension of flexibility. As a starting point, we concentrate on the power allocation related issues in this paper. Specifically, we develop an algorithm of power allocation for the duration-differentiated energy services with peer-to-peer charging. A given supply profile is shown to be adequate if and only if the proposed algorithm produces a feasible allocation. In the case of an inadequate supply, the algorithm can be modified to find the adequacy gap, i.e., the minimum additional purchase of power required to satisfy all the load requirements. We are also interested in the market implementation of such services and the practical implications that can be derived from the market analysis. This is under our current investigation.

The rest of this paper is organized as follows. The problem is formulated in Section II. The allocation algorithm and its connection to the adequacy is presented in Section III. A further analysis of the algorithm is presented in Section IV. The adequacy gap is discussed in Section V. Some illustrative examples are shown in Section VI. Finally, we conclude the paper and articulate the future work in Section VII. The notation adopted in this paper is standard, in general, and will be made clear as we proceed. The proofs are left out in consideration of fluency and page limitations. The ideas of the proofs are sketched and the details can be found in the longer version of the paper available from the authors.

II. PROBLEM FORMULATION

Suppose the power is delivered over a horizon of T time slots, indexed by $\{t : t = 1, 2, \ldots, T\}$. Denote by h_t the power available at the tth time slot. Consider a collection of N flexible loads, wherein load i is characterized by three parameters: the duration requirement r_i, the arrival time a_i, and the deadline d_i. In a compact form, load i is represented as a triple $\mathcal{L}_i = (r_i, a_i, d_i)$. We tacitly assume r_i, a_i, and d_i are all integers, and $r_i \leq d_i - a_i + 1$, i.e., the duration requirement should be no more than the number of time slots available between the arrival and deadline. Load i is said to be active at time t if $a_i \leq t \leq d_i$. An active load presents a three-state switching behavior: charging state, discharging state, and off state. For technical simplicity, assume that the charging and discharging rate of each load is 1 unit of power per time slot. As such, the power consumption of an active load i at time slot t is given by

$$u(i,t) = \begin{cases} 1, & \text{Charging state,} \\ -1, & \text{Discharging state,} \\ 0, & \text{Off state.} \end{cases}$$

Note that the acquired one unit of power in the charging state can come from either the allocation of the supply or the discharging of another load. For a load in the discharging state, there is another load in the charging state at the same time slot receiving the discharged power. A load in the off state gets neither charged nor discharged.

The energy state of load i at time slot t is then given by

$$s(i,t) := s(i,t-1) + u(i,t) = \sum_{k=a_i}^{t} u(i,k),$$

where a zero initial state is assumed, i.e., $s(i, a_i) = 0$. The energy state $s(i,t)$ signifies the level of accumulated energy in load i at time t. It is required to satisfy the constraint

$$s(i,t) \geq 0 \tag{1}$$

for every possible i and t. This constraint is due to the zero initial condition assumed, thus a load cannot discharge more units than it has obtained. In this paper, we assume that each load is equipped with an auxiliary power storage capacity so that it can get more units of power than required at the intermediate time slots.

Denote the *supply profile* and the *demand profile* by

$$\begin{aligned} \boldsymbol{h} &= \begin{bmatrix} h_1 & h_2 & \cdots & h_T \end{bmatrix}', \\ \boldsymbol{r} &= \begin{bmatrix} r_1 & r_2 & \cdots & r_N \end{bmatrix}', \end{aligned}$$

respectively. A supply profile is called adequate if there exists a three-state allocation such that all the load requirements are satisfied subject to the energy state constraint (1). If, further, the supply profile has no surplus after allocation, it is called exactly adequate.

As a first attempt, we consider the case where $\sum_{t=1}^{T} h_t = \sum_{i=1}^{N} r_i$ for simplicity. That is to say, the total energy given by the supply profile equals the total energy required by the demand. We begin with a collection of homogenous loads with the same arrival time $a_i = 1$ and the same deadline $d_i = T$, as in the duration-differentiated energy services.

Note that finding a three-state allocation is equivalent to filling a $(-1, 0, 1)$-matrix with prescribed row and column sums. Accordingly, the energy state constraint (1) corresponds to a constraint on the leading partial row sums of the $(-1, 0, 1)$-matrix. When the peer-to-peer charging is absent, the allocation reduces to a $(0, 1)$-matrix completion problem, as in [14], [16] and [20].

We firstly address the adequacy. Given a supply profile and a demand profile, can we find a way to verify whether the supply profile is adequate or not? If it is adequate, how do we find a feasible power allocation? A natural follow-up question is the adequacy gap. If the given supply profile is inadequate, what is the minimum amount of supplementary purchase required so as to make the total supply adequate? This amounts to solving the following optimization problem:

$$\begin{aligned} \min_{\boldsymbol{p}} \quad & \sum_{t=1}^{T} p_t, \\ \text{subject to} \quad & \boldsymbol{h} + \boldsymbol{p} \text{ is adequate,} \end{aligned} \tag{2}$$

where $\boldsymbol{p} = \begin{bmatrix} p_1 & p_2 & \cdots & p_T \end{bmatrix}'$ is an integer vector denoting the supplemental purchase.

III. ADEQUACY AND ALLOCATION ALGORITHM

In this section, we design a power allocation algorithm for the duration-differentiated energy services with peer-to-peer charging, and show that a given supply profile is adequate if and only if the algorithm gives a feasible allocation.

An allocation algorithm or policy is said to be *optimal* in terms of feasibility if for any adequate supply profile, it always gives a feasible allocation. In this sense, an optimal allocation algorithm can be used to check whether a supply profile is adequate or not. In what follows, such an optimal allocation algorithm is proposed. It runs backwardly from the last time slot to the preceding ones and, thus, requires the knowledge of the whole supply profile in advance, which can be obtained from a day-ahead forecast in a forward market.

To introduce the algorithm, we need more notation. Keep in mind that the allocation is performed in a backward way. A load is said to be urgent (insatiable, respectively) at time slot t if its remaining duration requirement is equal to (greater than, respectively) t. Intuitively, an urgent load at time slot t needs to receive power at every remaining time slot. Denote the set of urgent loads (insatiable loads, respectively) at time slot t by $\mathbb{U}_t$ ($\mathbb{I}_t$, respectively), i.e.,

$$\mathbb{U}_t = \{\mathcal{L}_i : r_i - \sum_{k=t+1}^{T} u(i,k) = t\},$$

$$\mathbb{I}_t = \{\mathcal{L}_i : r_i - \sum_{k=t+1}^{T} u(i,k) > t\}.$$

Moreover, denote the cardinality of $\mathbb{U}_t$ and $\mathbb{I}_t$ by $|\mathbb{U}_t|$ and $|\mathbb{I}_t|$, respectively.

The essential idea of our algorithm is a recursive implementation of the following three rules.

1: The supply h_t is allocated with priority given to the loads with longer remaining durations.

2: If the supply h_t is not sufficient to meet all the urgent loads at time t, i.e., $h_t < |\mathbb{U}_t|$, then the peer-to-peer charging is activated. Specifically, there would be $|\mathbb{U}_t| - h_t$ number of urgent loads to be charged by their non-urgent peers. The loads with shorter remaining durations are given priority to discharge power.

3: After the implementation of rule 1 and 2, if there are two loads in the off state with their remaining durations differing by more than one unit, then the load with the shorter duration conveys one unit of power to the one with the longer duration. Again, the priority of charging and discharging is consistent with rule 1 and 2.

We demonstrate these steps by a simple example. Suppose $T = 9$, $\boldsymbol{r} = [9\ 9\ 9\ 6\ 6\ 3\ 1]'$, and $h_T = 2$. Table I presents two possible allocations at time T following the above three rules. Note that the first three loads are urgent and the power supply is only enough to meet two of them. The other one needs to receive power from the non-urgent load with the shortest duration. Then, observe that the fifth and sixth loads have a duration difference greater than one. By rule 3, the sixth load conveys one unit of power to the fifth load. We consider the two allocations equivalent since one can be transformed into the other via load re-ordering.

(a)

$\diagdown$	$\cdots$	2
9	$\cdots$	1
9	$\cdots$	1
9	$\cdots$	1
6	$\cdots$	0
6	$\cdots$	1
3	$\cdots$	-1
1	$\cdots$	-1

(b)

$\diagdown$	$\cdots$	2
9	$\cdots$	1
9	$\cdots$	1
9	$\cdots$	1
6	$\cdots$	1
6	$\cdots$	0
3	$\cdots$	-1
1	$\cdots$	-1

TABLE I

THE TWO POSSIBLE LAST-COLUMN ALLOCATIONS

We are now in a position to formally present our allocation algorithm and its connection to the adequacy of a given supply profile. See Algorithm 1 and Theorem 3.1.

Algorithm 1 Allocation Algorithm

```
Input: The demand profile $\boldsymbol{r}$ and supply profile $\boldsymbol{h}$.
Output: An exception or an allocation matrix $U$.
1: Initialization: $t = T, U = 0_{N \times T}$;
2: Allocate the supply $h_t$ to renew $U(1:N,t)$ by the three steps described above;
3: Update the supply profile by deleting $h_t$. Update $r_i = r_i - U(i,t)$, for every $i = 1, 2, \ldots, N$;
4: If some load is insatiable under the new supply profile, this algorithm terminates with an exception. Otherwise, $t = t - 1$;
5: If $r$ becomes a zero vector, output $U$. Otherwise, go to step 2.
```

Theorem 3.1: The supply profile $\boldsymbol{h}$ is exactly adequate if and only if Algorithm 1 produces a feasible allocation matrix.

The sufficiency proof of Theorem 3.1 is straightforward. The main idea of necessity proof is sketched below. We first show that given an adequate supply profile, there always exists a feasible allocation, whose last-column states of all the loads satisfy the three rules above. Such a feasible allocation is called the last-column optimal allocation. Next, we show that such last-column optimal property still holds when the supply profile and demand profile are updated as in Step 3 of Algorithm 1. By applying the last-column optimal allocation repeatedly, Algorithm 1 generates a feasible allocation U^*. This allocation is referred to as a canonical allocation and satisfies the following property:

For every $t = 1, 2, \ldots, T$, the allocation represented by $U^*(1:N, 1:t)$ is last-column optimal with respect to the supply profile $\boldsymbol{h^t} = \begin{bmatrix} h_1 & h_2 & \cdots & h_t \end{bmatrix}'$ and the demand profile $\boldsymbol{r^t} = \begin{bmatrix} r_1^t & r_2^t & \cdots & r_N^t \end{bmatrix}'$, where r_i^t is the ith row sum of $U^*(1:N, 1:t)$.

Theorem 3.1 indicates that one can verify the adequacy of the supply profile via Algorithm 1. However, the algorithm does not give an explicit adequacy condition, as opposed to the majorization relation obtained in the case of no peer-to-peer charging [14], [21]. To obtain an analytic condition for the problem at hand, a technical issue is that the supply profile now cannot be arbitrarily reordered due to the energy state constraint (1). Nevertheless, one can still draw some inspirations from the majorization condition applicable to the case with no peer-to-peer charging. In fact, the majorization condition represents a certain dominating relation between two vectors derived from the supply profile and the demand profile, respectively. Likewise, we also expect to find a way to construct two corresponding vectors that satisfy a similar dominating relation in the case with peer-to-peer charging. Detailed discussions are out of the scope of this paper.

Before proceeding, note that the algorithm proposed above is non-causal since it requires the knowledge of the whole supply profile in advance. In real-time allocations, a causal algorithm is more desirable, in the sense that the allocation at time slot t does not require the information of future supply. It is of great interest to ask whether there exists a causal optimal allocation policy. If there exists, how to design one? If not, can we explore some sub-optimal heuristic allocation algorithms? These questions are under current investigation. We expect that the approach in [22] may give us a hint.

IV. ALGORITHM ANALYSIS

In this section, we further analyze Algorithm 1 and relate it to dynamic programming.

A. *Complexity of Algorithm 1*

The Algorithm 1 mainly involves a loop of at most T iterations. Each iteration consists of a column allocation, a profile update and an insatiable load detection, corresponding to Step 2, Step 3 and Step 4, respectively. Consequently, the complexity of each iteration is $\mathcal{O}(N)$. In a worst-case analysis, the overall complexity of Algorithm 1 is $\mathcal{O}(T \cdot N)$.

B. *Significance of the three rules*

In the duration-differentiated energy services with no peer-to-peer charging, the longest duration first (LDF) algorithm works as an optimal allocation policy. In essence, the LDF algorithm is nothing but the first rule. The necessity of the second rule is also easy to understand since urgent loads have to be duly served. If the urgent loads cannot be charged by the supplier, then other loads should discharge their energy storage to satisfy the requirements of the urgent loads. Inspired by the idea of LDF, the loads with shorter remaining durations discharge their energy with priority. To pursue an optimal allocation policy in terms of feasibility, the third rule is indispensable. Although it may involve some redundant peer-to-peer charging in some special cases, it is necessary to enforce this rule to guarantee a feasible allocation in general. This is illustrated by an example in Table II. When $t = 4$ in Algorithm 1, the first load is not urgent by definition, but the remaining durations of the two loads have a difference 2. According to the third rule, we should renew the allocation at the last time slot as $U(1:2,4) = [1 \ -1]'$. Otherwise, we cannot achieve a feasible allocation.

$\diagdown$	0	2	2	0
3	0	1	1	1
1	0	1	1	-1

TABLE II

C. *Relation to dynamic programming*

In view of the recursive nature of Algorithm 1, one can relate the algorithm to a dynamic programming.

Let us decompose the optimal feasible allocation problem into T stages. At stage k, the state S_k is a sub-allocation $U(1:N,(k+1):T)$ and the decision d_k is an allocation $U(1:N,k)$ of the supply h_k. The supplies from the first time slot to the kth time slot remain to be allocated. If there exists a feasible allocation with decision d_k and sub-allocation S_k, then the return $f_k(d_k, S_k)$ for stage k is one. Otherwise, the return is zero. Let $S_T = \emptyset$ and $d_0 = \emptyset$.

Consider the following optimization problem over the decision variables $d_T, d_{T-1}, \ldots, d_2, d_1$:

$$\max \left[f_1(d_1, S_1) + f_2(d_2, S_2) + \cdots + f_T(d_T, S_T)\right]$$

subject to

$$\begin{aligned} &S_{k-1} = [d_k \ S_k], \quad k = 1, 2, \ldots, T, \\ &d_k \in D_k, \quad k = 1, 2, \ldots, T, \end{aligned}$$

where D_k denotes the set of all N-dimensional $(-1,0,1)$-vectors with the sum of elements being smaller than or equal to h_k. Starting from the last stage, a dynamic programming gives an optimal solution $d_T, d_{T-1}, \ldots, d_1$ in a backward manner. It turns out that our algorithm can be regarded as an implementation of such a dynamic programming.

V. ADEQUACY GAP

In Section III, we give a way to verify the adequacy of the supply profile via Algorithm 1. In the event of an inadequate supply, the algorithm terminates with an exception. A natural follow-up question then arises: What is the minimum amount of additional power needed to serve all the loads? This boils down to the optimization problem (2) of which the optimal value is called adequacy gap, denoted by g. It turns out that such an adequacy gap problem can be solved via a slight modification of Algorithm 1.

Theorem 5.1: If the supply profile $\boldsymbol{h}$ is inadequate, then Algorithm 2 gives a feasible supplementary purchase $\boldsymbol{p}$ that achieves the adequacy gap g.

```
Algorithm 2 Adequacy Gap Algorithm
Input: The demand profile $\boldsymbol{r}$ and supply profile $\boldsymbol{h}$.
Output: A supplementary purchase $\boldsymbol{p}$ and adequacy gap $g$.
1: Initialization: $t=T, U=\mathbf{0}_{N\times T}, \ \boldsymbol{p}=0, \ g=0$;
2: Do as in Step 2 of Algorithm 1;
3: Let $p_t = |\mathbb{I}_{t-1}|$. Update $g = g + p_t$ and $h_t = h_t + p_t$;
4: Do as in Step 2 of Algorithm 1;
5: Do as in Step 3 of Algorithm 1. Update $t = t - 1$;
6: If $t == 0$, output $\boldsymbol{p}, g$. Otherwise, go to step 2.
```

It is not difficult to see the vector $\boldsymbol{p}$ given by Algorithm 2 is a feasible supplementary purchase vector. However, it will take more efforts to show that the sum of the elements of $\boldsymbol{p}$ indeed achieves the adequacy gap. Also note that the optimal solution of (2) may not be unique in general. This will be illustrated by a numerical example in the next section.

VI. NUMERICAL EXAMPLES

In this section, several numerical examples are given to demonstrate the duration-differentiated energy services with peer-to-peer charging.

Example 1: One significant characteristic of the duration-differentiated energy services is that the load is indifferent to the actual delivery time. Suppose that there is a load i requiring two units of power over six time slots. As shown in Table III, the first three serving strategies are indifferent for the load. However, the last serving strategy is not applicable, since the energy state $s(i,3)=-1$ violates the constraint (1).

1)	2	1	1	1	-1	0	0
2)	2	1	-1	0	1	0	1
3)	2	1	0	0	1	0	0
4)	2	1	-1	-1	1	1	1

TABLE III

Example 2: Suppose the power is delivered over eight time slots and there are six flexible loads. The supply profile is given by $\boldsymbol{h}=[6\ 5\ 1\ 4\ 4\ 3\ 2\ 1]'$ and the demand profile is given by $\boldsymbol{r}=[8\ 8\ 4\ 3\ 2\ 1]'$. The following table shows the process of implementation of Algorithm 1. In the case when multiple loads require the same remaining duration, we allocate in a way such that the remaining demand profile after the allocation is still a non-increasing vector. One can see that the algorithm produces a feasible allocation and thus the supply profile $\boldsymbol{h}$ is adequate.

╲	⋯	1
8	⋯	1
8	⋯	1
4	⋯	1
3	⋯	0
2	⋯	-1
1	⋯	-1

$\Longrightarrow$

╲	⋯	2	1
7	⋯	1	1
7	⋯	1	1
3	⋯	0	1
3	⋯	0	0
3	⋯	0	-1
2	⋯	0	-1

$\Longrightarrow$

╲	⋯	3	2	1
6	⋯	1	1	1
6	⋯	1	1	1
3	⋯	0	0	1
3	⋯	0	0	0
3	⋯	1	0	-1
2	⋯	0	0	-1

$\Rightarrow$

╲	⋯	4	3	2	1
5	⋯	1	1	1	1
5	⋯	1	1	1	1
3	⋯	1	0	0	1
3	⋯	1	0	0	0
2	⋯	0	1	0	-1
2	⋯	0	0	0	-1

$\Longrightarrow$

╲	⋯	4	4	3	2	1
4	⋯	1	1	1	1	1
4	⋯	1	1	1	1	1
2	⋯	0	1	0	0	1
2	⋯	0	1	0	0	0
2	⋯	1	0	1	0	-1
2	⋯	1	0	0	0	-1

$\Rightarrow$

╲	⋯	1	4	4	3	2	1
3	⋯	1	1	1	1	1	1
3	⋯	1	1	1	1	1	1
2	⋯	0	0	1	0	0	1
2	⋯	0	0	1	0	0	0
1	⋯	-1	1	0	1	0	-1
1	⋯	0	1	0	0	0	-1

$\Rightarrow$

╲	⋯	5	1	4	4	3	2	1
2	⋯	1	1	1	1	1	1	1
2	⋯	1	1	1	1	1	1	1
2	⋯	1	0	0	1	0	0	1
2	⋯	1	0	0	1	0	0	0
2	⋯	1	-1	1	0	1	0	-1
1	⋯	0	0	1	0	0	0	-1

$\Rightarrow$

╲	6	5	1	4	4	3	2	1
1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1
1	1	1	0	0	1	0	0	1
1	1	1	0	0	1	0	0	0
1	1	1	-1	1	0	1	0	-1
1	1	0	0	1	0	0	0	-1

$\Rightarrow$

╲	6	5	1	4	4	3	2	1
0	1	1	1	1	1	1	1	1
0	1	1	1	1	1	1	1	1
0	1	1	0	0	1	0	0	1
0	1	1	0	0	1	0	0	0
0	1	1	-1	1	0	1	0	-1
0	1	0	0	1	0	0	0	-1

TABLE IV

Specifically, the supply profile is specified in the first row of the table, while the remaining demand profile is specified in the first column. The allocation process is demonstrated by the $(-1,0,1)$-matrices in the right bottom corner. Note that we have not explicitly specified the loads charged through peer-to-peer charging. In fact, at a specific time slot, every load in the charging state can be a candidate charged by the loads in the discharging state.

It is worth mentioning that the supply and demand profiles in the above example do not satisfy the majorization condition as in [14], [21]. That is to say, the supply profile $\boldsymbol{h}=[6\ 5\ 1\ 4\ 4\ 3\ 2\ 1]'$ is inadequate to meet the demand profile $\boldsymbol{r}=[8\ 8\ 4\ 3\ 2\ 1]'$, if the peer-to-peer charging functionality is not allowed. This echoes the motivating question raised in the beginning of this paper. The introduction of peer-to-peer charging does bring in a new advantage. It enlarges the set of adequate supply profiles and thus eases the use of the renewable generation.

Example 3: Consider an example related to an inadequate supply profile. Suppose the power is delivered over six time slots and there are five flexible loads. The supply profile is $\boldsymbol{h}=[5\ 2\ 1\ 1\ 2\ 1]'$ and the demand profile is $\boldsymbol{r}=[6\ 6\ 3\ 1\ 1]'$. The following table shows the process of implementation of Algorithm 2. The algorithm produces a feasible supplementary purchase vector $\boldsymbol{p}=[0\ 3\ 1\ 1\ 0\ 0]'$ that achieves the adequacy gap $g=5$.

╲	⋯	1
6	⋯	1
6	⋯	1
3	⋯	1
1	⋯	-1
1	⋯	-1

$\Longrightarrow$

╲	⋯	2	1
5	⋯	1	1
5	⋯	1	1
2	⋯	0	1
2	⋯	0	-1
2	⋯	0	-1

$\Longrightarrow$

╲	⋯	1	2	1
4	⋯	0	1	1
4	⋯	1	1	1
2	⋯	0	0	1
2	⋯	0	0	-1
2	⋯	0	0	-1

$\Rightarrow$

╲	⋯	1	2	1
4	⋯	0	1	1
4	⋯	1	1	1
2	⋯	0	0	1
2	⋯	0	0	-1
2	⋯	0	0	-1

$\Longrightarrow$

╲	⋯	**2**	2	1
4	⋯	1	1	1
4	⋯	1	1	1
2	⋯	0	0	1
2	⋯	0	0	-1
2	⋯	0	0	-1

$\Rightarrow$

╲	⋯	1	**2**	2	1
3	⋯	0	1	1	1
3	⋯	1	1	1	1
2	⋯	0	0	0	1
2	⋯	0	0	0	-1
2	⋯	0	0	0	-1

$\Longrightarrow$

╲	⋯	**2**	**2**	2	1
3	⋯	1	1	1	1
3	⋯	1	1	1	1
2	⋯	0	0	0	1
2	⋯	0	0	0	-1
2	⋯	0	0	0	-1

$\Rightarrow$

╲	⋯	2	**2**	**2**	2	1
2	⋯	0	1	1	1	1
2	⋯	0	1	1	1	1
2	⋯	0	0	0	0	1
2	⋯	1	0	0	0	-1
2	⋯	1	0	0	0	-1

$\Longrightarrow$

╲	⋯	**5**	**2**	**2**	2	1
2	⋯	1	1	1	1	1
2	⋯	1	1	1	1	1
2	⋯	1	0	0	0	1
2	⋯	1	0	0	0	-1
2	⋯	1	0	0	0	-1

$\Rightarrow$

╲	5	**5**	**2**	**2**	2	1
1	1	1	1	1	1	1
1	1	1	1	1	1	1
1	1	1	0	0	0	1
1	1	1	0	0	0	-1
1	1	1	0	0	0	-1

$\Longrightarrow$

╲	5	**5**	**2**	**2**	2	1
0	1	1	1	1	1	1
0	1	1	1	1	1	1
0	1	1	0	0	0	1
0	1	1	0	0	0	-1
0	1	1	0	0	0	-1

TABLE V

As mentioned before, there may be other solutions to the optimization problem (2) other than the one indicated by our algorithm. For example, the supplementary purchase vectors

$\tilde{\boldsymbol{p}}_{\mathbf{1}} = [0\ 1\ 1\ 1\ 2\ 0]'$ and $\tilde{\boldsymbol{p}}_{\mathbf{2}} = [0\ 0\ 0\ 1\ 0\ 4]'$ also achieve the adequacy gap 5, as shown in the next table.

╲	5	**3**	**2**	**2**	**4**	1
6	1	1	1	1	1	1
6	1	1	1	1	1	1
3	1	1	0	0	0	1
1	1	0	0	0	1	-1
1	1	0	0	0	1	-1

╲	5	2	1	**2**	2	**5**
6	1	1	1	1	1	1
6	1	1	1	1	1	1
3	1	0	1	0	0	1
1	1	0	-1	0	0	1
1	1	0	-1	0	0	1

TABLE VI

VII. CONCLUSION AND FUTURE WORK

In this paper, we introduce the peer-to-peer charging to the duration-differentiated energy services. This new dimension of load flexility enlarges the set of adequate supply profiles and, thus, eases the demand/supply balance in a power grid with high renewables. An algorithm is developed for power allocation with peer-to-peer charging allowed. Moreover, we show that the supply profile is adequate if and only if the algorithm gives a feasible allocation. Numerical examples are used to illustrate some characteristics of the differentiated energy services and the effectiveness of our algorithms.

In the future, we wish to explore whether there exists a causal optimal policy, which is more desirable with regard to real-time implementations. We shall further examine the adequacy with the hope of finding an explicit characterization of the adequate supply profiles. We also wish to design a market mechanism so as to motivate consumers to involve themselves in such differentiated energy services.

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