Networked Stabilization of Multi-input Systems over Shared Channels with Scheduling/Control Co-design ⋆

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Abstract

In this paper, we study the networked stabilization of a continuous-time multi-input system wherein the multiple control inputs are transmitted through a small number of shared channels with stochastic multiplicative uncertainties. Transmission scheduling over the shared channels needs to be performed so that at any time instant, each channel transmits only one control input. The main novelty of this work lies in the idea of scheduling/control co-design which suggests that the design of the transmission scheduling and the controller should be treated jointly rather than separately. By virtue of such a co-design, a sufficient condition is obtained for the channels' overall quality of service required for stabilization given in terms of twice of the topological entropy of the plant. A numerical example is provided to illustrate our result.

Key words: Networked control system; networked stabilization; scheduling/control co-design; topological entropy.

1 Introduction

The rapid development of communication technology is transforming more and more feedback control systems into networked control systems (NCSs), in which the information exchange between the controller and plant is through communication networks. In the study of NCSs, one fundamental issue, among many others, is networked stabilization with various information constraints.

A well-known result developed in the past two decades is the data rate theorem or capacity theorem for networked stabilization of single-input systems. It reveals that the minimum data rate (channel capacity) rendering stabilization possible for a single-input system is given by the topological entropy of the plant, i.e., the logarithm of the absolute product of unstable poles for a discrete-time plant, or the sum of unstable poles for a continuous-time plant. This result has been established under a variety of information constraints, see [10,12] for quantization, [3] for signal-to-noise ratio constraint, [9,32] for stochastic multiplicative uncertainty, to name just a few.

For stabilizing multi-input NCSs, references [23,31,32] exploit the idea of channel/controller co-design and show that the minimum total channel capacity required for stabilization is given again in terms of the topological entropy of the plant. An alternative approach is developed in [25,28] to study the discrete-time MIMO systems over signal-to-noise ratio constrained channels via utilizing a coding strategy. Recently, the stabilization of discrete-time MIMO systems over stochastic multiplicative noise channels has been discussed in [22].

Researchers have also showed interest in the scenario when multiple sensors and/or multiple actuators need to share a communication channel. A dynamic scheduling and stochastic scheduling policy have been proposed in [18,19] respectively, wherein a predictive control method has been utilized along with the scheduling scheme. A stochastic scheduling policy has also been developed in [14] in studying an estimation problem over a shared channel. The authors in [15] derived maximally allowable transmission interval and maximally allowable delay that guarantee stability of NCSs with only partial sensor measurements and partial actuator signals granted access to the channel per sampling interval.

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Inspired by the existing works, we study in this paper the networked stabilization of multi-input systems over shared channels with stochastic multiplicative uncertainties. Transmission scheduling of the control inputs is performed over the shared channels so that at any time instant, each channel serves only one control input. Our objective is to derive a condition on the quality of service of the channels rendering stabilization possible.

We start with a simple case when there is only one shared channel and then extend to the general case when there are several shared channels. The key for establishing our result lies in the idea of scheduling/control co-design, i.e., the transmission scheduling is assumed to be designed simultaneously with the controller design. By virtue of this additional design freedom, a sufficient condition is obtained for the overall quality of service of the channels required for stabilization given in terms of twice of the topological entropy of the plant.

Some preliminary results on the special case of a single shared channel have been reported in the authors' earlier conference paper [6]. In the current paper, we extend the study to the general case of multiple shared channels. Furthermore, we provide an alternative design scheme with uniform scheduling and linear coding that has not been considered in the conference version. To ensure the completeness of the story, the case of one shared channel is also included in the current paper. The concept of scheduling/control co-design has been used in the study of embedded control systems for integrating control and computing [30]. Recently, it attracts interests from the research community of NCSs as well [2,4,7,18].

The rest of this paper is organized as follows. Section 2 formulates the problem. Section 3 studies the networked stabilization over one single shared channel. Section 4 extends the study to the general case of several shared channels. The problem is revisited in Section 5, where an alternative scheme with uniform scheduling and linear coding is presented. An illustrative example is worked out in Section 6. The paper is concluded in Section 7.

Most notation in this paper is more or less standard and will be made clear as we proceed. The symbol $\odot$ means Hadamard product. Denote the identity matrix by I, the open unit disk by $\mathbb{D}$, and the open left half complex plane by $\mathbb{C}^-$. Denote by $\mathcal{S}_n$ the space of $n \times n$ real symmetric matrices. Denote by $\mathcal{S}_n^{\mathbb{C}}$ the complexification of $\mathcal{S}_n$, i.e., the space of $n \times n$ complex symmetric matrices. The spectrum of a linear operator $\mathscr{L}$ from $\mathcal{S}_n$ to $\mathcal{S}_n$ is defined to be $\sigma(\mathscr{L}) = \{\lambda \in \mathbb{C} : \exists X \in \mathcal{S}_n^{\mathbb{C}}, X \neq 0, \mathscr{L}(X) = \lambda X\}$. For a stable $m \times m$ transfer function $G(s)$, define the mixed norm

$$\|G(s)\|_{2,1} = \sqrt{\left\| \left[\|G(s)_{ij}\|_2^2 \right] \right\|_1} = \sqrt{\max_{1 \le j \le m} \sum_{i=1}^{m} \|G(s)_{ij}\|_2^2},$$

where $\|G(s)_{ij}\|_2$ is the $\mathcal{H}_2$ norm of the (i,j)th entry of $G(s)$.

2 Problem Formulation

Consider the NCS as shown in Fig. 1, where $[A|B]$ is a continuous-time linear time invariant (LTI) system described by the state space model

$$\dot{x}(t) = Ax(t) + Bu(t), \quad x(0) = x_0,$$

with $A \in \mathbb{R}^{n \times n}, B \in \mathbb{R}^{n \times m}$. Assume that $[A|B]$ is stabilizable and the state $x(t)$ is available for feedback. The control signal $v(t) = Fx(t)$, where F is a state feedback gain, is transmitted through a communication network to the plant.

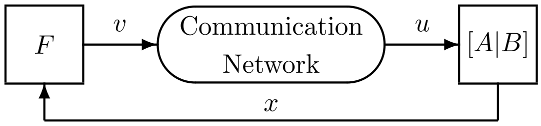

Fig. 1. State feedback via communication network.

For a single-input system, by nature, one communication channel suffices to serve the transmission purpose. For a multi-input system, a common setup in many existing studies [23,31,32] is to assume the same number of communication channels as the number of control inputs so that each channel transmits a particular control input.

Here, we are interested in a scenario when the control inputs have to share a small number of communication channels. We start with a simple case when there is only one communication channel serving all control signals. In this case, a multiplexer/de-multiplexer pair can be exploited so that only one control input is transmitted though the channel at one time, as depicted in Fig. 2. The task performed by the multiplexer/de-multiplexer pair is referred to as transmission scheduling, which is reminiscent of the time-division-multiple-access (TDMA) scheme [13] developed in the communication field.

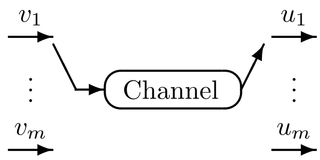

Fig. 2. A multiplexer channel.

Due to the uncertainties in the communication process, the control signal received is a distorted version of the transmitted one. We focus on the stochastic multiplicative uncertainty in the transmission process, and model the input-output relationship of the channel as

$$r(t) = \kappa(t)s(t), \tag{1}$$

where $s(t)$ is the transmitted signal, $r(t)$ is the received signal and $\kappa(t)$ is a white noise process independent of $s(t)$. Let the mean and autocovariance of $\kappa(t)$ be μ and $\sigma^2\delta(\tau)$, respectively. We can rewrite (1) into $r(t) = r_1(t)+r_2(t)$ with $r_1(t) = \mu s(t)$ and $r_2(t) = (\kappa(t)-\mu)s(t)$. Then, $r_1(t)$ is simply a constant scaled version of the transmitted signal, while $r_2(t)$ is the transmission noise. The following quantity (referred to as quality of service of the channel)

$$\text{QoS} = \frac{\boldsymbol{E}[r_1(t)^2]}{\boldsymbol{E}[r_2(t)^2]} = \frac{\mu^2\boldsymbol{E}[s(t)^2]}{\sigma^2\boldsymbol{E}[s(t)^2]} = \frac{\mu^2}{\sigma^2}$$

plays an important role in our study.

The above networked multi-input system over a shared channel can be considered as a special type of switched linear system called multiple controller system [26] as below:

$$\dot{x}(t) = Ax(t) + B_\theta\kappa(t)v_\theta(t), \tag{2}$$

where θ is the abbreviation for the scheduling signal $\theta(t)$ that is piecewise constant and takes values from the index set $\mathcal{I} = \{1, 2, \ldots, m\}$. Assume that $\theta(t) = i$ for $t \in [t_1, t_2)$, then $B_\theta = B_i$ and $v_\theta(t) = F_i x(t)$ for $t \in [t_1, t_2)$, where B_i is the ith column of B and F_i is the ith row of F.

Note that periodic scheduling is popular in engineering design and implementation from the practical perspective. Also, as stated in Theorem 3.11 in [26], a switched linear system can be stabilized if and only if it can be stabilized with a periodic scheduling signal. Therefore, hereinafter we consider the case of periodic scheduling in the sense that there exists a period T such that $\theta(t+T) = \theta(t), \forall t \geq 0$. Without loss of generality, assume that the control inputs are sequentially transmitted from the first input to the last input. Denote $\pi = \begin{bmatrix}\pi_1 & \pi_2 & \ldots & \pi_m\end{bmatrix}'$ as a probability vector, where $0 \leq \pi_i \leq 1, \sum_{i=1}^m \pi_i = 1$. Then a periodic scheduling signal can be expressed as

$$\theta(t) = \begin{cases} 1, & \text{if } t \in [kT, kT+\pi_1 T), \\ 2, & \text{if } t \in [kT+\pi_1 T, kT+(\pi_1+\pi_2)T), \\ \vdots & \\ m, & \text{if } t \in [kT+(\sum_{i=1}^{m-1}\pi_i)T, (k+1)T), \end{cases} \tag{3}$$

where $k = 0, 1, 2, \ldots$. Intuitively, π_i represents the duty cycle of the ith control input, i.e., the fraction of transmission time allocated to the ith control input. The vector π is also referred to as an allocation vector in the sequel.

We are interested in finding a condition on the channel's quality of service under which the networked stabilization can be achieved by a linear state feedback controller $v(t) = Fx(t)$. The main novelty of this work is in the idea of scheduling/control co-design, i.e., designing the scheduling signal and the controller simultaneously.

Before proceeding, recall that the topological entropy [1] of a matrix $A \in \mathbb{R}^{n\times n}$ is given by $h(A) = \sum_{|\lambda_i|>1} \ln|\lambda_i|$, where λ_i are the eigenvalues of A. Based on this, we define the topological entropy of a continuous-time system $\dot{x}(t) = Ax(t)$ as $H(A) = h(e^A) = \sum_{\Re(\lambda_i)>0}\lambda_i$, where λ_i are the eigenvalues of A and $\Re(\lambda_i)$ means the real part of λ_i.

3 Stabilization over a Single Shared Channel

We first define the notion of MS stabilizability.

Definition 1 $[A|B]$ *is said to be MS stabilizable over a shared channel with stochastic multiplicative uncertainty if there exist* $\theta(t)$ *and* F *such that for every initial state* $x(0)$, $N(t) \triangleq \boldsymbol{E}[x(t)x'(t)]$ *is well-defined for any* $t > 0$ *and* $\lim_{t\to\infty} N(t) = 0$.

As mentioned before, the multi-input NCS over a shared channel can be considered as a switched linear system given by (2). With the periodic scheduling signal $\theta(t)$ as in (3), applying Itô's formula [17] to $N(t)$ yields

$$\dot{N}(t) = \mathscr{L}_i(N(t)), \text{ if } t \in \left[kT+\sum_{l=1}^{i-1}\pi_l T, kT+\sum_{l=1}^{i}\pi_l T\right), \tag{4}$$

where $i = 1, 2, \ldots, m$, $k = 0, 1, 2, \ldots$, and $\mathscr{L}_i$ is a linear operator from $\mathcal{S}_n$ to $\mathcal{S}_n$ given by

$$\mathscr{L}_i : X \mapsto (A+\mu B_i F_i)X + X(A+\mu B_i F_i)' + \sigma^2 B_i F_i X F_i' B_i'.$$

Integrating both sides of (4) and discretizing $N(t)$ with period T yields

$$N((k+1)T) = \mathscr{T}(N(kT)), k = 0, 1, 2, \ldots,$$

where $\mathscr{T}$ is a linear operator from $\mathcal{S}_n$ to $\mathcal{S}_n$ given by

$$\mathscr{T} = e^{\pi_m\mathscr{L}_m T}e^{\pi_{m-1}\mathscr{L}_{m-1}T}\ldots e^{\pi_1\mathscr{L}_1 T}. \tag{5}$$

The operator $\mathscr{T}$ is called stable if $\sigma(\mathscr{T}) \subset \mathbb{D}$. One can easily verify that $N(t) \to 0$ when $t \to \infty$ is equivalent to $N(kT) \to 0$ when $k \to \infty$. Hence, the stabilization is accomplished if and only if $\sigma(\mathscr{T}) \subset \mathbb{D}$.

To analyze the operator $\mathscr{T}$, we exploit the Campbell-Baker-Hausdorff (CBH) formula [8] that goes as follows: There exists $\epsilon > 0$ such that for $t \in (-\epsilon, \epsilon)$, there holds

$$e^{\mathcal{A}t}e^{\mathcal{B}t} = e^{(\mathcal{A}+\mathcal{B})t+\frac{1}{2}[\mathcal{A},\mathcal{B}]t^2+\frac{1}{12}([\mathcal{A},[\mathcal{A},\mathcal{B}]]+[\mathcal{B},[\mathcal{B},\mathcal{A}]])t^3+\cdots},$$

where $[\mathcal{A}, \mathcal{B}] = \mathcal{A}\mathcal{B} - \mathcal{B}\mathcal{A}$ is the commutator product of $\mathcal{A}$ and $\mathcal{B}$. Based on the CBH formula, a method called average method [26] is developed in the switched system theory in studying the stabilization of switched linear systems. For the problem at hand, the following lemma can be easily deduced with the CBH formula. The details of the proof are omitted for brevity. Similar results can be found in [26,27].

Lemma 1 *Let $\mathcal{A}_1, \ldots, \mathcal{A}_m$ be linear operators from $\mathcal{S}_n$ to $\mathcal{S}_n$. Then there exists $\epsilon > 0$ such that when $0 < t < \epsilon$, it holds $e^{\mathcal{A}_m t} e^{\mathcal{A}_{m-1} t} \ldots e^{\mathcal{A}_1 t} = e^{(\sum_{i=1}^m \mathcal{A}_i)t + o(t)}$, where $\frac{o(t)}{t} \to 0$ as $t \to 0$.*

Applying Lemma 1 to the operator $\mathscr{T}$ as in (5) yields

$$\mathscr{T} = e^{(\sum_{i=1}^m \pi_i \mathscr{L}_i)T + o(T)} \tag{6}$$

for sufficiently small T. In this case, one can approximate the logarithm of the operator $\mathscr{T}$ by the product of T and an average operator $\mathscr{L} = \sum_{i=1}^m \pi_i \mathscr{L}_i$. Hence, the networked stabilization problem amounts to studying $\mathscr{L}$ that is precisely a linear operator from $\mathcal{S}_n$ to $\mathcal{S}_n$ given by

$$\begin{aligned}\mathscr{L} : X \mapsto (A+BMF)X + X(A+BMF)' \\ + B(\Sigma^2 \odot (FXF'))B',\end{aligned}$$

where

$$\begin{aligned}M &= \text{diag}\{\pi_1\mu, \pi_2\mu, \ldots, \pi_m\mu\}, \\ \Sigma^2 &= \text{diag}\{\pi_1\sigma^2, \pi_2\sigma^2, \ldots, \pi_m\sigma^2\}.\end{aligned}$$

The operator $\mathscr{L}$ is called stable if $\sigma(\mathscr{L}) \subset \mathbb{C}^-$.

The following lemma gives several criteria in verifying the stability of $\mathscr{L}$. The equivalence between statements (a), (b), and (c) can be referred to [32]. The equivalence between the implications (a) and (d) is the continuous-time counterpart of Theorem 6.4 in [9]. The details of the proof are omitted here for brevity.

Lemma 2 *The following statements are equivalent:*

(a) $\sigma(\mathscr{L}) \subset \mathbb{C}^-$.
(b) *There exists* $X > 0$ *and* F *such that* $\mathscr{L}(X) < 0$.
(c) *There exists* $X > 0$ *such that*

$$A'X + XA - XBM(\Sigma^2 \odot (B'XB))^{-1} MB'X < 0.$$

(d) *It holds*

$$\inf_{D \in \mathcal{D}, F: A+BMF \text{ is stable}} \|D^{-1} T(s) D \Phi\|_{2,1} < 1, \tag{7}$$

where $T(s) = F(sI - A - BMF)^{-1} BM$, $\Phi = M^{-1}\Sigma$, *and* $\mathcal{D}$ *is the set of all* $m \times m$ *positive diagonal matrices.*

We assume that the allocation vector π can be designed simultaneously with the controller. How to choose the allocation vector π appropriately so as to facilitate the stabilization? Some inspiration can be drawn from the Wonham decomposition of a system that was originally put forward in [29] for the multi-input pole placement problem. Given a multi-input stabilizable system $[A|B]$, one can carry out a series of controllable/uncontrollable decompositions which eventually give rise to the Wonham decomposition as follows:

$$\left[\begin{bmatrix} A_1 & * & \cdots & * \\ 0 & A_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & * \\ 0 & \cdots & 0 & A_m \end{bmatrix} \middle| \begin{bmatrix} b_1 & * & \cdots & * \\ 0 & b_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & * \\ 0 & \cdots & 0 & b_m \end{bmatrix} \right], \tag{8}$$

where $[A_i|b_i]$ is stabilizable for $i = 1, 2, \ldots, m$. It is clear that the ith control input is used to stabilize the ith subsystem $[A_i|b_i]$. Moreover, previous experience [9,32] tells that there is a minimum amount of stabilization work that has to be done by the ith input which is given in terms of $H(A_i)$. With this observation in mind, we design the duty cycle of the ith control input to be equal to the ratio of $H(A_i)$ over $H(A)$, i.e., $\pi_i = \frac{H(A_i)}{H(A)}$. It turns out that such design of allocation vector fits the purpose. A condition on the channel's quality of service required for stabilization is presented in the following theorem which has been shown in the conference version [6]. We include the proof here for the completeness of the paper.

Theorem 1 *The system $[A|B]$ is MS stabilizable over a shared channel with stochastic multiplicative uncertainty if* $\text{QoS} > 2H(A)$.

PROOF. Without loss of generality, we assume that all the eigenvalues of A lie in the open right half complex plane. To show the theorem, we aim to find a positive diagonal matrix D, a state feedback gain F together with an allocation vector π such that the inequality (7) holds. If that is the case, by Lemma 2, $\sigma(\mathscr{L}) \subset \mathbb{C}^-$. Then, in view of (6), one can always choose T sufficiently small to make $\sigma(\mathscr{T}) \subset \mathbb{D}$ and thus achieve the MS stabilization. In the sequel, the desired matrices D, F and the allocation vector π are constructed.

Without loss of generality, $[A|B]$ is assumed to be of the form given by the Wonham decomposition as in (8), where each subsystem $[A_i|b_i]$ is stabilizable with state dimension n_i. Clearly, we have $\sum_{i=1}^m n_i = n$. For each subsystem $[A_i|b_i]$, it has been shown that [3]

$$\inf_{f_i: A_i + b_i \pi_i \mu f_i \text{ is stable}} \|T_i(s)\|_2^2 = 2H(A_i), \tag{9}$$

where

$$T_i(s) = f_i(sI - A_i - b_i\pi_i\mu f_i)^{-1}b_i\pi_i\mu. \quad (10)$$

We now set

$$D = \text{diag}\{1, \epsilon, \ldots, \epsilon^{m-1}\} \quad (11)$$

with ϵ a small positive real number. Also define

$$P = \text{diag}\{I_{n_1}, \epsilon I_{n_2}, \ldots, \epsilon^{m-1} I_{n_m}\}.$$

Then

$$\begin{aligned} &D^{-1}T(s)D\Phi \\ &= D^{-1}F(sI-A-BMF)^{-1}BMD\Phi \\ &= D^{-1}FP(sI-P^{-1}AP-P^{-1}BMFP)^{-1}P^{-1}BMD\Phi \\ &= F(sI-P^{-1}AP-P^{-1}BMDF)^{-1}P^{-1}BMD\Phi. \end{aligned} \quad (12)$$

Simple calculations show that

$$P^{-1}AP = \begin{bmatrix} A_1 & O(\epsilon) & \cdots & O(\epsilon) \\ 0 & A_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & O(\epsilon) \\ 0 & \cdots & 0 & A_m \end{bmatrix}, \quad (13)$$

$$P^{-1}BMD = \begin{bmatrix} b_1\pi_1\mu & O(\epsilon) & \cdots & O(\epsilon) \\ 0 & b_2\pi_2\mu & \ddots & \vdots \\ \vdots & \ddots & \ddots & O(\epsilon) \\ 0 & \cdots & 0 & b_m\pi_m\mu \end{bmatrix}, \quad (14)$$

where $\frac{O(\epsilon)}{\epsilon}$ approaches to a finite constant as $\epsilon \to 0$. By making ϵ small, we approximately decompose $[A|BM]$ into m single-input systems $[A_i|b_i\pi_i\mu], i = 1, 2, \ldots, m$. Since QoS $> 2H(A)$ and $H(A) = \sum_{i=1}^m H(A_i)$, we can choose $\pi_i = \frac{H(A_i)}{H(A)}$ satisfying $\sum_{i=1}^m \pi_i = 1$ and $\pi_i\text{QoS} > 2H(A_i)$. We now set $F = \text{diag}\{f_1, f_2, \ldots, f_m\}$ such that $A_i + b_i\pi_i\mu f_i$ is stable and $\|T_i(s)\|_2^2 < \pi_i\text{QoS}$, where $T_i(s)$ is given by (10). The existence of such f_i is guaranteed by (9) and the fact that $\pi_i\text{QoS} > 2H(A_i)$. In view of (12), (13) and (14), it can now be verified that

$$\begin{aligned} &D^{-1}T(s)D\Phi \\ &= \text{diag}\{\frac{T_1(s)}{\sqrt{\pi_1\text{QoS}}}, \frac{T_2(s)}{\sqrt{\pi_2\text{QoS}}}, \ldots, \frac{T_m(s)}{\sqrt{\pi_m\text{QoS}}}\} + O(\epsilon; s), \end{aligned}$$

where $O(\epsilon; s) \to 0$ as $\epsilon \to 0$. Since $\|T_i(s)\|_2 < \sqrt{\pi_i\text{QoS}}$, it follows that $\|D^{-1}T(s)D\Phi\|_{2,1} < 1$ for sufficiently small ϵ. This completes the proof.

Remark 1 *As in the above proof, a feasible allocation is to make $\pi_i\text{QoS} > 2H(A_i)$ and thus is not unique. Taking $\pi_i = \frac{H(A_i)}{H(A)}$ is only one of the feasible allocations. Such allocation of transmission time shares a similar spirit of the channel resource allocation as in [23,31,32]. In those works, the capacities are allocated among the input channels subject to a total capacity constraint, while in this work, communication resource is allocated by deciding the duty cycle of each control input.*

Remark 2 *One can see that fast switching is needed to accomplish stabilization when the QoS is close to $2H(A)$. In fact, there is a trade-off between the QoS and the allowable switching rate. The larger QoS, the lower switching rate can be adopted. The allowable switching rate also depends on the controller. The design of f_i as in the above proof minimizes $\|T_i(s)\|_2$ and has a positive role in permitting a lower switching rate. For a detailed estimation of the allowable switching rate, one can refer to [11,21].*

We wish to mention that the authors' conference paper [6] claimed that QoS $> 2H(A)$ is necessary and sufficient for MS stabilization but the necessity proof therein was not correct. Whether this condition is necessary remains to be answered and is under our current investigation.

4 Stabilization over Several Shared Channels

The same idea can extend to the more general scenario when multiple control inputs are transmitted through several shared channels. Consider again the continuous-time linear system $[A|B]$ with control inputs generated by a state feedback controller $v(t) = Fx(t)$. Assume that there are p communication channels available between the controller and the actuators. Each channel has its input-output relationship modeled by

$$r_j(t) = \kappa_j(t)s_j(t), j = 1, 2, \ldots, p,$$

where $s_j(t)$ is transmitted signal, $r_j(t)$ is received signal, and $\kappa_j(t)$ is white noise with mean μ_j and autocovariance $\sigma_j^2\delta(\tau)$. Assume that $\kappa_j(t), j = 1, 2, \ldots, p$, are mutually independent and also independent of $s_j(t)$. The quality of service of the channels are given by

$$\text{QoS}_j = \frac{\mu_j^2}{\sigma_j^2}, j = 1, 2, \ldots, p.$$

The overall quality of service is then given by $\text{QoS}_{\text{overall}} = \sum_{j=1}^p \text{QoS}_j$. Each communication channel is equipped with a multiplexer/de-multiplexer pair so that transmission scheduling of the control inputs is performed over the shared channels.

Different from the single-channel case, we introduce a scaling factor $\alpha_j, j = 1, 2, \ldots, p$, for each channel, i.e., the control signals are scaled first by a factor of α_j before being transmitted over the jth channel. The scaling factors can be designed. Then, the networked system can be described by the following dynamic equation:

$$\dot{x}(t) = Ax(t) + \begin{bmatrix} B'_{\theta_1} \\ B'_{\theta_2} \\ \vdots \\ B'_{\theta_p} \end{bmatrix}' \begin{bmatrix} \kappa_1(t) & 0 & \cdots & 0 \\ 0 & \kappa_2(t) & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \kappa_p(t) \end{bmatrix} \begin{bmatrix} \alpha_1 v_{\theta_1}(t) \\ \alpha_2 v_{\theta_2}(t) \\ \vdots \\ \alpha_p v_{\theta_p}(t) \end{bmatrix},$$

where $\theta_j, j = 1, 2, \ldots, p$, is the abbreviation for the scheduling signal $\theta_j(t)$ associated with the jth communication channel. As before, the scheduling signals are piecewise constant and take values from the index set $\mathcal{I} = \{1, 2, \ldots, m\}$.

The MS stabilizability defined in Section 3 can be adapted directly to the case considered here. Again, periodic scheduling is adopted with the scheduling period denoted by T. Each channel is associated with a probability vector $\pi^j = \begin{bmatrix} \pi_1^j & \pi_2^j & \ldots & \pi_m^j \end{bmatrix}'$, $j = 1, 2, \ldots, p$, where the non-negative component π_i^j represents the duty cycle of the ith control input over the jth channel and satisfies the constraint $\sum_{i=1}^m \pi_i^j = 1$. Without loss of generality, assume that the scheduling signal $\theta_j(t), j = 1, 2, \ldots, p$, takes the following form:

$$\theta_j(t) = \begin{cases} 1, & \text{if } t \in [kT, kT + \pi_1^j T), \\ 2, & \text{if } t \in [kT + \pi_1^j T, kT + (\pi_1^j + \pi_2^j)T), \\ \vdots & \\ m, & \text{if } t \in [kT + (\sum_{i=1}^{m-1} \pi_i^j)T, (k+1)T), \end{cases}$$

where $k = 0, 1, 2, \ldots$.

Now, let

$$\tilde{\mu}_i = \sum_{j=1}^p \pi_i^j \alpha_j \mu_j, \quad \tilde{\sigma}_i^2 = \sum_{j=1}^p \pi_i^j \alpha_j^2 \sigma_j^2, \quad i = 1, 2, \ldots, m.$$

Mimicking the analysis in the previous section on the behavior of the state covariance, we will end up with the following linear operator from $\mathcal{S}_n$ to $\mathcal{S}_n$:

$$\tilde{\mathscr{L}} : X \mapsto (A + B\tilde{M}F)X + X(A + B\tilde{M}F)' + B(\tilde{\Sigma}^2 \odot (FXF'))B',$$

where

$$\tilde{M} = \mathrm{diag}\{\tilde{\mu}_1, \tilde{\mu}_2, \ldots, \tilde{\mu}_m\}, \tilde{\Sigma}^2 = \mathrm{diag}\{\tilde{\sigma}_1^2, \tilde{\sigma}_2^2, \ldots, \tilde{\sigma}_m^2\}.$$

In analogy to the previous study, when the scheduling period T is sufficiently small, the MS stabilization can be accomplished if there exists an F together with a set of allocation vectors $\pi^1, \pi^2, \ldots, \pi^p$ and scaling factors $\alpha_1, \alpha_2, \ldots, \alpha_p$ such that the operator $\tilde{\mathscr{L}}$ is stable, i.e., $\sigma(\tilde{\mathscr{L}}) \subset \mathbb{C}^-$. A straightforward adaption of Lemma 2 gives several criteria in verifying the stability of $\tilde{\mathscr{L}}$.

We wish to find a condition on the channels' quality of service under which the networked stabilization can be achieved. Again, how to design the transmission scheduling over the shared channels is a critical issue. It may happen that several channels participate in transmitting the same control input. In other words, several channels may contribute partially to the transmission of a control input. Considering this, analogously to the case of $p = 1$, here we design the allocation vectors $\pi^1, \pi^2, \ldots, \pi^p$ to be such that $\sum_{j=1}^p \pi_i^j \mathrm{QoS}_j > 2H(A_i)$ for $i = 1, 2, \ldots, m$. We end up with a condition on the overall quality of service rendering stabilization possible. The distribution of the individual channel's quality of service appears not relevant. See the following theorem.

Theorem 2 $[A|B]$ *is MS stabilizable over several shared channels with stochastic multiplicative uncertainties if* $\mathrm{QoS}_{\mathrm{overall}} > 2H(A)$.

PROOF. Without loss of generality, assume that all the eigenvalues of A lie in the open right half complex plane and $[A|B]$ is in the form of Wonham decomposition given by (8). In light of an adaption of the implication (d) in Lemma 2, it suffices to show

$$\inf_{D \in \mathcal{D}, F: A + B\tilde{M}F \text{ is stable}} \|D^{-1}\tilde{T}(s)D\tilde{\Phi}\|_{2,1} < 1,$$

where $\tilde{T}(s) = F(sI - A - B\tilde{M}F)^{-1}B\tilde{M}$, $\tilde{\Phi} = \tilde{M}^{-1}\tilde{\Sigma}$, and $\mathcal{D}$ is the set of all $m \times m$ positive diagonal matrices. We have three design freedoms here: the allocation vectors π^j, the scaling factors α_j, and the state feedback gain F.

Since $\mathrm{QoS}_{\mathrm{overall}} > 2H(A)$, we can find a set of allocation vectors $\pi^1, \pi^2, \ldots, \pi^p$ such that $\sum_{j=1}^p \pi_i^j \mathrm{QoS}_j > 2H(A_i)$ for all $i = 1, 2, \ldots, m$. Also, we can design scaling factors α_j such that $\frac{\mu_1}{\alpha_1\sigma_1^2} = \frac{\mu_2}{\alpha_2\sigma_2^2} = \cdots = \frac{\mu_p}{\alpha_p\sigma_p^2}$. In this case, one can verify that $\sum_{j=1}^p \pi_i^j \mathrm{QoS}_j = \frac{\tilde{\mu}_i^2}{\tilde{\sigma}_i^2}$ and thus, $\frac{\tilde{\mu}_i^2}{\tilde{\sigma}_i^2} > 2H(A_i)$.

Now, for each subsystem $[A_i|b_i]$ in the Wonham decomposition, we have [3]

$$\inf_{f_i: A_i + b_i\tilde{\mu}_i f_i \text{ is stable}} \|\tilde{T}_i(s)\|_2^2 = 2H(A_i), \tag{15}$$

where

$$\tilde{T}_i(s) = f_i(sI - A_i - b_i\tilde{\mu}_i f_i)^{-1} b_i\tilde{\mu}_i. \tag{16}$$

Let $F = \mathrm{diag}\{f_1, f_2, \ldots, f_m\}$ such that $A_i + b_i\tilde{\mu}_i f_i$ is stable and $\|\tilde{T}_i(s)\|_2^2 < \frac{\tilde{\mu}_i^2}{\tilde{\sigma}_i^2}$, where $\tilde{T}_i(s)$ is given by (16). The existence of such f_i is guaranteed by (15) and the inequality $\frac{\tilde{\mu}_i^2}{\tilde{\sigma}_i^2} > 2H(A_i)$. Let D be as in (11). In a similar way to the proof of Theorem 1, one can verify that

$$\begin{aligned} &D^{-1}\tilde{T}(s)D\tilde{\Phi} \\ &= \mathrm{diag}\{\tilde{T}_1(s)\frac{\tilde{\sigma}_1}{\tilde{\mu}_1}, \tilde{T}_2(s)\frac{\tilde{\sigma}_2}{\tilde{\mu}_2}, \ldots, \tilde{T}_m(s)\frac{\tilde{\sigma}_m}{\tilde{\mu}_m}\} + O(\epsilon; s), \end{aligned}$$

where $O(\epsilon; s) \to 0$ as $\epsilon \to 0$. Since $\|\tilde{T}_i(s)\|_2 < \frac{\tilde{\mu}_i}{\tilde{\sigma}_i}$, it follows that $\|D^{-1}\tilde{T}(s)D\tilde{\Phi}\|_{2,1} < 1$ for sufficiently small ϵ. This completes the proof.

5 Stabilization with Uniform Scheduling and Linear Coding

In this section, we present an alternative scheme with uniform scheduling and linear coding. For simplicity, we focus on the case of a single shared channel. The extension to the multiple shared channels is straightforward.

Recall in Section 3, the main idea of scheduling/control co-design is to perform the Wonham decomposition (8) and assign the duty cycle of the ith control input to be $\frac{H(A_i)}{H(A)}$, i.e., proportional to the topological entropy of the ith subsystem $[A_i|b_i]$. It is interesting to see if it is possible to stabilize the networked control system with a simple uniform scheduling, i.e., assign equal duty cycle to all the control inputs. It turns out the answer is affirmative, provided that a linear encoding/decoding mechanism is allowed.

As shown in Fig. 3, the control signal v undergoes a linear encoding via a matrix $S \in \mathbb{R}^{m\times m}$ before it is transmitted over the shared channel and the received signal p undergoes a linear decoding via a matrix $R \in \mathbb{R}^{m\times m}$ before it is applied to the actuators. The encoder matrix S and decoder matrix R can be freely designed. Let the scheduling signal $\theta(t)$ be as in (3) with $\pi_1 = \pi_2 = \cdots = \pi_m = \frac{1}{m}$, i.e., uniform scheduling is adopted.

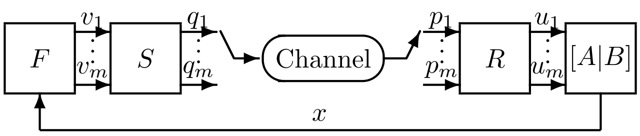

Fig. 3. Networked control with linear coding.

Mimicking the analysis in Section 3 on the behavior of the state covariance, we end up with the following linear operator from $\mathcal{S}_n$ to $\mathcal{S}_n$:

$$\begin{aligned} \overline{\mathscr{L}} : X \mapsto (A+BR\overline{M}SF)X + X(A+BR\overline{M}SF)' \\ + BR(\overline{\Sigma}^2 \odot (SFXF'S'))R'B', \end{aligned}$$

where $\overline{M} = \dfrac{\mu}{m}I$ and $\overline{\Sigma}^2 = \dfrac{\sigma^2}{m}I$. In analogy to previous analysis, when the scheduling period T is sufficiently small, the MS stabilization can be accomplished if there exists a state feedback gain F together with an encoder matrix S and a decoder matrix R such that the operator $\overline{\mathscr{L}}$ is stable.

Theorem 3 *When uniform scheduling is used,* $[A|B]$ *is MS stabilizable via linear coding if* $\mathrm{QoS} > 2H(A)$.

To prepare the proof of Theorem 3, we introduce the notion of majorization and a useful lemma.

For $x, y \in \mathbb{R}^n$, we denote by $x^\downarrow$ and $y^\downarrow$ the rearranged versions of x and y so that their elements are arranged in a non-increasing order. We say that x is majorized by y, denoted by $x \preccurlyeq y$, if

$$\begin{cases} \displaystyle\sum_{i=1}^{k} x_i^\downarrow \le \sum_{i=1}^{k} y_i^\downarrow, & \text{for } k = 1, \ldots, n-1, \\ \displaystyle\sum_{i=1}^{n} x_i^\downarrow = \sum_{i=1}^{n} y_i^\downarrow. \end{cases}$$

Majorization orders the level of fluctuations when the averages are the same. Specifically, $x \preccurlyeq y$ indicates that the elements of x are more even or, less spread out, than the elements of y.

Lemma 3 ([20]) *There exists a real symmetric matrix* X *with eigenvalues* $\lambda_1, \lambda_2, \ldots, \lambda_n$*, and diagonal elements* $d_1, d_2, \ldots, d_n$*, if and only if*

$$\begin{bmatrix} d_1 & d_2 & \cdots & d_n \end{bmatrix}' \preccurlyeq \begin{bmatrix} \lambda_1 & \lambda_2 & \cdots & \lambda_n \end{bmatrix}'.$$

When the majorization condition in Lemma 3 is satisfied, algorithms for finding the desired symmetric matrix X have also been developed in the literature [5,16].

PROOF of Theorem 3. We seek a state feedback gain F together with an encoder/decoder pair such that

$$\inf_{D\in\mathcal{D}, F: A+BR\overline{M}SF \text{ is stable}} \|D^{-1}\overline{T}(s)D\overline{\Phi}\|_{2,1} < 1,$$

holds, where $\overline{T}(s) = SF(sI - A - BR\overline{M}SF)^{-1}BR\overline{M}$, $\overline{\Phi} = \overline{M}^{-1}\overline{\Sigma} = \sqrt{m}\dfrac{\sigma}{\mu}I$.

To this end, without loss of generality, assume that $[A|B]$ is in the form of Wonham decomposition (8). We design F in the same way as in the proof to Theorem 1 and let $S = UD^{-1}, R = DU'$, where $U \in \mathbb{R}^{m\times m}$ is an orthogonal matrix to be designed and D is in the form of (11). Then

$$\begin{aligned}
&\|\overline{T}(s)\overline{\Phi}\|_{2,1}^2 \\
&= \max_{1\le j\le m} \frac{1}{2\pi}\int_{-\infty}^{\infty}\left\{\overline{\Phi}\overline{T}^*(j\omega)T(j\omega)\overline{\Phi}\right\}_{jj} d\omega \\
&= \max_{1\le j\le m} \{\overline{\Phi}U(\mathrm{diag}\{\|T_1(s)\|_2^2,\ldots,\|T_m(s)\|_2^2\} \\
&\qquad\qquad + o(\epsilon))U'\overline{\Phi}\}_{jj} \\
&= \max_{1\le j\le m} \{\overline{\Phi}U(\mathrm{diag}\{2H(A_1),\ldots,2H(A_m)\} \\
&\qquad\qquad + o(\epsilon))U'\overline{\Phi}\}_{jj},
\end{aligned}$$

where $T_i(s)$ is given by (10) with $\pi_i = \frac{1}{m}$. Since

$$\begin{bmatrix}\frac{H(A)}{m} & \frac{H(A)}{m} & \ldots & \frac{H(A)}{m}\end{bmatrix}' \preccurlyeq \begin{bmatrix}H(A_1) & H(A_2) & \ldots & H(A_m)\end{bmatrix}',$$

by Lemma 3, an orthogonal matrix U can be constructed such that $(U(\mathrm{diag}\{H(A_1),\ldots,H(A_m)\})U')_{jj} = \frac{H(A)}{m}$ holds for $j = 1,2,\ldots,m$. Then $\|\overline{T}(s)\overline{\Phi}\|_{2,1}^2 = \frac{2H(A)}{\mathrm{QoS}} + o(\epsilon)$. Since QoS $> 2H(A)$, when ϵ is sufficiently small, we have $\|\overline{T}(s)\overline{\Phi}\|_{2,1} < 1$ which completes the proof.

6 An Illustrative Example

Consider the following unstable system $[A|B]$:

$$A=\begin{bmatrix}4&0&0&0\\0&2&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}, \quad B=\begin{bmatrix}B_1 & B_2 & B_3\end{bmatrix}=\begin{bmatrix}1&0&0\\1&1&0\\0&1&1\\0&0&1\end{bmatrix},$$

with $x_0 = \begin{bmatrix}1 & 1 & 1 & 1\end{bmatrix}'$. It is stabilizable and already in the Wonham decomposition form with

$$A = \mathrm{diag}\{A_1, A_2, A_3\}, \quad b_1 = \begin{bmatrix}1 & 1\end{bmatrix}', \quad b_2 = 1, \quad b_3 = 1,$$

where $A_1 = \begin{bmatrix}4&0\\0&2\end{bmatrix}$, $A_2 = 1, A_3 = 1$. The topological entropy of the plant is $H(A) = H(A_1) + H(A_2) + H(A_3) = (4+2)+1+1 = 8$.

For illustration, we consider a case when there is only one channel linking the controller and the actuators. Let $\mu = 4$, $\sigma^2 = 0.98$. The quality of service of the channel is QoS $= \frac{\mu^2}{\sigma^2} = 16.32$ which is greater than $2H(A)$ by two percents. Theorem 1 implies that in this case, the NCS can be MS stabilized by a feasible scheduling/control co-design. One such co-design is carried out as below.

Design the periodic scheduling signal $\theta(t)$ as in (3) with $T = 0.1$ (sec) and $\pi = \begin{bmatrix}0.75 & 0.125 & 0.125\end{bmatrix}'$. For the controller design, we solve the $\mathcal{H}_2$ optimal $T_i(s)$ as in (9) for the following three single-input systems:

$$\begin{aligned}
[A_1|b_1\pi_1\mu] &= \left[\begin{array}{cc|c}4&0&3\\0&2&3\end{array}\right], \\
[A_2|b_2\pi_2\mu] &= \left[\begin{array}{c|c}1&0.5\end{array}\right], \\
[A_3|b_3\pi_3\mu] &= \left[\begin{array}{c|c}1&0.5\end{array}\right],
\end{aligned}$$

yielding the optimal feedback gains $f_1 = \begin{bmatrix}-8 & 4\end{bmatrix}$, $f_2 = -4$, and $f_3 = -4$, respectively. Let $F = \mathrm{diag}\{f_1, f_2, f_3\}$. With this scheduling/control co-design, the Frobenius norm of the state covariance $N(kT)$ converges to zero asymptotically, as shown in Fig. 4.

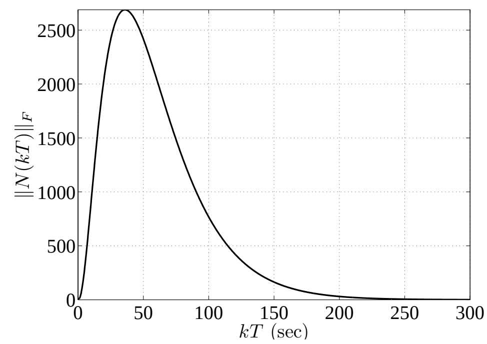

Fig. 4. Closed-loop evolution of $\|N(kT)\|_F$.

7 Conclusion

In this paper, we study the stabilization of a continuous-time networked multi-input system wherein the control inputs are transmitted through several shared channels subject to stochastic multiplicative uncertainties. Our investigation starts with a simple case when there is only one shared channel and then extends to the general case of multiple shared channels. The key idea lies in the introduction of scheduling/control co-design, i.e., design the transmission scheduling and the controller jointly to stabilize the NCS. Under such co-design, a sufficient condition on the channels' quality of service required for stabilization is obtained, given in terms of twice of the topological entropy of the plant.

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References

[1] R. Bowen, "Entropy for group endomorphisms and homogeneous space", *Trans. of the American Mathematical Society*, vol. 153, pp. 401-414, Jan. 1971.

[2] M. S. Branicky, S. M. Phillips, and W. Zhang, "Scheduling and feedback co-design for networked control systems", in *Proc. 41st IEEE Conf. Decision Contr.*, pp. 1211-1217, Las Vegas, Nevada USA, 2002.

[3] J. H. Braslavsky, R. H. Middleton, and J. S. Freudenberg, "Feedback stabilization over signal-to-noise ratio constrained channels", *IEEE Trans. Automat. Contr.*, vol. 52, pp. 1391-1403, Aug. 2007.

[4] R. Brockett, "Stabilization of motor networks", in *Proc. 34st IEEE Conf. Decision Contr.*, pp. 1484-1488, New Orleans, LA, 1995.

[5] N. N. Chan and K.-H. Li, "Diagonal elements and eigenvalues of a real symmetric matrix", *J. Math. Anal. Appl.*, vol. 91, pp. 562-566, 1983.

[6] W. Chen, J. Yao, and L. Qiu, "Stabilization of networked multi-input systems over a shared bus with scheduling/control co-design", in *Proc. 52nd IEEE Conf. Decision Contr.*, pp. 7235-7240, 2013.

[7] S.-L. Dai, H. Lin, and S. S. Ge, "Scheduling-and-control codesign for a collection of networked control systems with uncertain delays", *IEEE Trans. Contr. Syst. Technol.*, vol. 18, pp. 66-78, Jan. 2010.

[8] H. D. Fegan, *Introduction to Compact Lie Groups*, World Scientific, 1991.

[9] N. Elia, "Remote stabilization over fading channels", *Systems & Control Letters*, vol. 54, no. 3, pp. 237-249, 2005.

[10] N. Elia and S. K. Mitter, "Stabilization of linear systems with limited information", *IEEE Trans. Automat. Contr.*, vol. 46, pp. 1384-1400, Sept. 2001.

[11] J. Ezzine and A. H. Haddad, "Error bounds in the averaging of hybrid systems", *IEEE Trans. Automat. Contr.*, vol. 34, pp. 1188-1192, Nov. 1989.

[12] M. Fu and L. Xie, "The sector bound approach to quantized feedback control", *IEEE Trans. Automat. Contr.*, vol. 50, pp. 1698-1711, Nov. 2005.

[13] A. Goldsmith, *Wireless Communications*, Cambridge University Press, 2005.

[14] V. Gupta, T. H. Chung, B. Hassibi, and R. M. Murray, "On a stochastic sensor selection algorithm with applications in sensor scheduling and sensor coverage", *Automatica*, vol. 42, no. 2, pp. 251-260, 2006.

[15] W. P. M. H. Heemels, A. R. Teel, N. van de Wouw, and D. Něsić, "Networked control systems with communication constraints: Tradeoffs between transmission intervals, delays and performance", *IEEE Trans. Automat. Contr.*, vol. 55, pp. 1781-1796, 2010.

[16] E. L. Isaacson, "Solution to similarity and the diagonal of a matrix", *Amer. Math. Monthly*, vol. 87, pp. 62-63, 1980.

[17] F. C. Klebaner, *Introduction to Stochastic Calculus with Applications*, (3rd ed.), Imperial College Press, London, 2012.

[18] M. Lješnjanin, D. E. Quevedo, and D. Něsić, "Packetized MPC with dynamic scheduling constraints and bounded packet dropouts", *Automatica*, vol. 50, no. 3, pp. 784-797, 2014.

[19] Y. Long, S. Liu, and L. Xie, "Stochastic channel allocation for networked control systems", *IEEE Control Systems Letters*, vol. 1, no. 1, pp. 176-181, 2017.

[20] A. W. Marshall, I. Olkin, and B. C. Arnold, *Inequalities: Theorey of Majorization and Its Applications*, 2nd ed., Springer, New York, 2011.

[21] M. Porfiri, D. G. Roberson, and D. J. Stilwell, "Fast switching analysis of linear switched systems using exponential splitting", *SIAM J. Control Optim.*, vol. 47, pp. 2582-2597, 2008.

[22] T. Qi, J. Chen, W. Su, and M. Fu, "Control under stochastic multiplicative uncertainties: Part I, fundamental conditions of stabilizability", *IEEE Trans. Automat. Contr.*, vol. 62, pp. 1269-1284, Mar. 2017.

[23] L. Qiu, G. Gu, and W. Chen, "Stabilization of networked multi-input systems with channel resource allocation", *IEEE Trans. Automat. Contr.*, vol. 58, pp. 554-568, Mar. 2013.

[24] A. Sahai and S. Mitter, "The necessity and sufficiency of anytime capacity for stabilization of a linear system over a noisy communication link-part I: scalar systems", *IEEE Trans. Inf. Theory*, vol. 52, pp. 3369-3395, Aug. 2006.

[25] Z. Shu and R. H. Middleton, "Stabilization over power-constrained parallel Gaussian channels", *IEEE Trans. Automat. Contr.*, vol. 56, pp. 1718-1724, July. 2011.

[26] Z. Sun and S. S. Ge, *Switched Linear Systems: Control and Design*, Springer-Verlag London, 2005.

[27] J. Tokarzewski, "Stability of periodically switched linear systems and the switching frequency", *Int. J. System Sci.*, vol. 18, pp. 697-726, 1987.

[28] F. J. Vargas, J. Chen, and E. I. Silva, "On stabilizability of MIMO systems over parallel noisy channels", in *Proc. 53rd IEEE Conf. Decision Contr.*, pp. 6074-6079, Los Angeles, USA, 2014.

[29] W. M. Wonham, "On pole assignment in multi-input controllable linear systems", *IEEE Trans. Automat. Contr.*, vol. AC-12, pp. 660-665, Dec. 1967.

[30] F. Xia and Y. Sun, *Control and Scheduling Codesign: Flexible Resource Management in Real-Time Control Systems*, Springer Berlin Heidelberg, 2008.

[31] N. Xiao, L. Xie, and L. Qiu, "Feedback stabilization of discrete-time networked systems over fading channels", *IEEE Trans. Automat. Contr.*, vol. 57, pp. 2176-2189, Sept. 2012.

[32] N. Xiao and L. Xie, "Feedback stabilization over stochastic multiplicative input channels: continuous-time case", in *Proc. Int. Conf. Contr., Automation, Robotics and Vision*, pp. 543-548, Singapore, 2010.