Stabilization of Cascaded Two-Port Networked Systems with Simultaneous Nonlinear Uncertainties ⋆

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Abstract

We investigate the robust stability of a networked control system (NCS) subject to simultaneous nonlinear uncertainties. An NCS is described as a feedback interconnection of a plant and a controller communicating through a bidirectional channel modelled by cascaded nonlinear two-port networks. This model is sufficiently rich to capture various properties of a real-world communication channel, such as distortion, interference, and nonlinearity. We provide a necessary and sufficient condition for the robust finite-gain stability of such an NCS when the model uncertainties in the plant and controller are measured by the gap metric and those in the nonlinear channels are measured by operator norms of the uncertain elements. This condition is given by an inequality involving "arcsine" of the uncertainty bounds and is derived from novel geometric insights underlying the robustness of a standard closed-loop system in the presence of conelike nonlinear perturbations on the system graphs.

Key words: uncertain systems, two-port networks, gap metric, nonlinear uncertainty, uncertainty quartets, robust stabilization.

1 Introduction

Feedback is widely used for handling uncertainties in the area of systems and control. Within a feedback loop, communication between the plant and controller plays an important role in that the achieved control performance and robustness heavily rely on the quality of communication. Most control systems can be regarded as structured networks with signals transmitted through channels powered by various devices, such as sensors or satellites. This gives rise to networked control systems (NCSs), which differ from standard closed-loop systems in that the information is exchanged through communication networks (Zhang, Branicky, & Phillips, 2001). The presence of non-ideal communication may introduce disturbances to a control system and hence significantly compromise its performance.

In this study, we introduce a two-port NCS model, generalizing a standard finite dimensional linear time-invariant (FDLTI) closed-loop system (Fig. 1) to a feedback system with cascaded two-port connections (Fig. 2). Therein, the plant P and controller C are uncertain FDLTI systems that may be open-loop unstable while the perturbations on the transmission matrices $\boldsymbol{T}_k$ of the two-port communication networks are nonlinear. It is known that model uncertainties are well characterized through the gap metric and its variants, among which the gap (Zames & El-sakkary, 1980; Georgiou, 1988; Georgiou & Smith, 1990; Qiu & Davison, 1992a), the pointwise gap (Schumacher, 1992; Qiu & Davison, 1992b) and the ν-gap (Vinnicombe, 1993, 2000) have been studied intensively. In this paper, we adopt the gap metric as our main analysis tool. Since the gap metric and its variants are topologically equivalent on the class of FDLTI systems, most of the results in this paper hold true for the ν-gap and the pointwise gap with similar arguments. As for the non-ideal two-port communication channels, we model their transmission matrices as $\boldsymbol{T} = \boldsymbol{I} + \boldsymbol{\Delta}$, where $\boldsymbol{\Delta}$ is a bounded nonlinear operator. It is noteworthy that the NCS framework introduced in this paper can also accommodate certain nonlinear plants and controllers. In particular, a nonlinear plant or controller may be modelled as the interconnection of an FDLTI system

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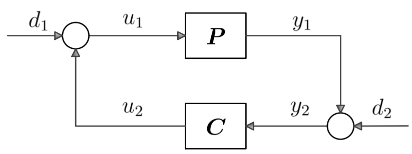

Fig. 1. A standard closed-loop system.

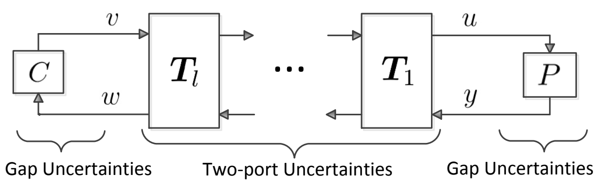

Fig. 2. Two-port NCS with uncertainties.

and a nonlinear communication channel. For the ease of subsequent presentation, however, we adopt the convention of calling the two FDLTI components in a two-port NCS the plant and controller. While it is well known that an uncertain system is usually represented by a fixed linear fractional transformation (LFT) of its uncertain component (Kimura, 1996; Zhou & Doyle, 1998), our two-port uncertainty model can be described by an uncertain LFT of a fixed component.

The main contribution of this study is a clean result for concluding the finite-gain stability of an NCS with different types and multiple sources of nonlinear uncertainties. A necessary and sufficient robust stability condition for such an NCS is then obtained. Based on the condition, a special robust stability margin of the NCS is introduced in terms of the Gang of Four transfer matrix (Åström & Murray, 2008). As for the synthesis of an optimally robust controller in the sense that the stability margin is maximized, it suffices to solve a one-block $\mathcal{H}_\infty$ optimization problem (Georgiou & Smith, 1990).

It is worth noting that similar geometric approaches to analyzing robust stability of nonlinear feedback systems have been developed from different perspectives; e.g., see (Teel, 1996) for uncertainty with conic interpretation and (Megretski & Rantzer, 1997; Cantoni, Jönsson, & Kao, 2012) for uncertainty subject to integral quadratic constraints. On the other hand, there have been relevant works on robust stabilization of NCSs with special architectures and various uncertainty; see (Zhao, Qiu, & Gu, 2020) for a detailed introduction of literature. A previous study by the authors in (Zhao, Khong, & Qiu, 2017) considers a two-port NCS involving only nonlinear channel uncertainties under a rather strong assumption. This study differs from or generalizes the previous results in that it handles the model uncertainty and the nonlinear perturbations within two-port communication channels simultaneously.

The rest of the paper is organized as follows. First in Section 2, we introduce open-loop & closed-loop systems, gap-type model uncertainties, and a preliminary robust stability result. Then in Section 3, we present our main result of the robust stability condition for two-port NCSs. The proof of the main theorem is provided in Section 4. In Section 5, we conclude this study and point out possible directions for future research.

2 Preliminaries

2.1 Open-Loop Stability

Let $\mathbb{F} = \mathbb{R}$ or $\mathbb{C}$ be the real or complex field, and $\mathbb{F}^n$ be the linear space of n-tuples of $\mathbb{F}$ over the field $\mathbb{F}$. For $x \in \mathbb{F}^n$, its Euclidean norm is denoted by $|x|$. For a complex number $s \in \mathbb{C}$, its real and imaginary parts are denoted by $\operatorname{Re} s$ and $\operatorname{Im} s$, respectively. Denote by $\mathcal{H}_\infty^{p\times m}$ the Hardy ∞-space of functions that are holomorphic and uniformly bounded on the right-half complex plane. This space is equipped with the $\mathcal{H}_\infty$ norm $\|G\|_\infty := \sup_{\operatorname{Re} s>0} \bar{\sigma}(G(s))$ for $G \in \mathcal{H}_\infty^{p\times m}$, where $\bar{\sigma}(\cdot)$ denotes the largest singular value of a complex matrix. An FDLTI system with transfer matrix G is said to be stable if $G \in \mathcal{H}_\infty^{p\times m}$. Denote by $\mathcal{P}^{p\times m}$ the set of all real rational proper transfer matrices, and by $\mathcal{RH}_\infty^{p\times m}$ the set of all real rational members in $\mathcal{H}_\infty^{p\times m}$.

Denote the set of all causal energy-bounded signals by

$$\mathcal{L}_2^n = \left\{ u : [0,\infty) \to \mathbb{R}^n : \ \|u\|_2^2 := \int_0^\infty |u(t)|^2\, dt < \infty \right\}.$$

For $T \geq 0$, define the truncation operator $\mathbf{\Gamma}_T$ on all signals $u : [0,\infty) \to \mathbb{R}^n$ by $(\mathbf{\Gamma}_T u)(t) = u(t)$ for $0 \leq t \leq T$ and $(\mathbf{\Gamma}_T u)(t) = 0$, otherwise.

Denote the extended $\mathcal{L}_2$ space (Willems, 1971, Chapter 2), (Feintuch & Saeks, 1982, Chapter 8) by

$$\mathcal{L}_{2e}^n := \{u : [0,\infty) \to \mathbb{R}^n : \ \mathbf{\Gamma}_T u \in \mathcal{L}_2^n, \ \forall\, T > 0\}.$$

A nonlinear system is represented by an operator

$$\boldsymbol{P} : \ \mathcal{L}_{2e}^m \to \mathcal{L}_{2e}^p.$$

We define the $\mathcal{L}_2$ domain of $\boldsymbol{P}$ as the set of all its input signals in $\mathcal{L}_2^m$ such that the output signals are in $\mathcal{L}_2^p$, i.e.,

$$\mathcal{D}(\boldsymbol{P}) := \{u \in \mathcal{L}_2^m : \ \boldsymbol{P}u \in \mathcal{L}_2^p\},$$

and correspondingly its $\mathcal{L}_2$ range as $\mathcal{R}(\boldsymbol{P}) := \boldsymbol{P}\mathcal{D}(\boldsymbol{P})$. A physical system should additionally be causal (Willems, 1971, Chapters 2 and 4), (Vidyasagar, 1993, Chapter 6), which is defined as follows.

Definition 1 *A system $\boldsymbol{P}$ is said to be causal if for all $T > 0$ and $u_1, u_2 \in \mathcal{L}_{2e}^m$,*

$$\mathbf{\Gamma}_T u_1 = \mathbf{\Gamma}_T u_2 \ \Rightarrow \ \mathbf{\Gamma}_T \boldsymbol{P} u_1 = \mathbf{\Gamma}_T \boldsymbol{P} u_2.$$

It is said to be strongly causal if it is causal and if for all $T > 0$, $\epsilon > 0$, and $T' \in (0, T]$, there exists a real number $\Delta_T > 0$ such that for any $u_1, u_2 \in \mathcal{L}_{2e}^m$,

$$\begin{aligned}&\boldsymbol{\Gamma}_{T'} u_1 = \boldsymbol{\Gamma}_{T'} u_2 \Rightarrow \\ &\qquad \|\boldsymbol{\Gamma}_{T'+\Delta_T}(\boldsymbol{P}u_1 - \boldsymbol{P}u_2)\| \leq \epsilon \|\boldsymbol{\Gamma}_{T+\Delta_T}(u_1 - u_2)\|.\end{aligned}$$

Throughout this study, we assume every system $\boldsymbol{P}$ is causal and that it has zero output whenever the input is zero, i.e., $\boldsymbol{P}0 = 0$. When $\boldsymbol{P} : \mathcal{L}_{2e}^m \to \mathcal{L}_{2e}^p$ is an FDLTI system, its restriction $\boldsymbol{P}|_{\mathcal{D}(\boldsymbol{P})} : \mathcal{D}(\boldsymbol{P}) \subset \mathcal{L}_2^m \to \mathcal{L}_2^p$ is equivalent, via the Fourier transform, to multiplication by a real rational proper transfer matrix in the frequency domain. On the other hand, an FDLTI system represented by a (possibly unbounded) linear operator $\boldsymbol{P} : \mathcal{D}(\boldsymbol{P}) \to \mathcal{L}_2^p$ can be uniquely and causally extended to an operator mapping from $\mathcal{L}_{2e}^m$ to $\mathcal{L}_{2e}^p$ (Georgiou & Smith, 1993, Proposition 11). Henceforth, we use $P \in \mathcal{P}^{p \times m}$ to denote the transfer matrix corresponding to such a $\boldsymbol{P}$, and we do not distinguish between an FDLTI system and its transfer matrix when the context is clear.

The finite-gain stability of such a system is defined as follows (Vidyasagar, 1993, Chapter 6).

Definition 2 *A system $\boldsymbol{P}$ is said to be (finite-gain) stable if there exists $\alpha > 0$ such that*

$$\|\boldsymbol{\Gamma}_T \boldsymbol{P} u\|_2 \leq \alpha \|\boldsymbol{\Gamma}_T u\|_2, \ \forall\, T \geq 0,\ u \in \mathcal{L}_{2e}^m. \qquad (1)$$

The following lemma is a direct consequence of (van der Schaft, 2017, Proposition 1.2.3).

Lemma 1 *A system $\boldsymbol{P}$ is finite-gain stable if and only if $\mathcal{D}(\boldsymbol{P}) = \mathcal{L}_2^m$ and*

$$\|\boldsymbol{P}\| := \sup_{0 \neq x \in \mathcal{L}_2^m} \frac{\|\boldsymbol{P}x\|_2}{\|x\|_2} < \infty.$$

The incremental stability (or Lipschitz continuity) of a system is defined as follows (Willems, 1971, Chapter 2).

Definition 3 *A system $\boldsymbol{P}$ is said to be incrementally stable if there exists a Lipschitz constant $L > 0$ such that*

$$\|\boldsymbol{\Gamma}_T(\boldsymbol{P}x - \boldsymbol{P}y)\|_2 \leq L \|\boldsymbol{\Gamma}_T(x - y)\|_2, \ \forall\, T \geq 0,\ x, y \in \mathcal{L}_{2e}^m.$$

Clearly, the incremental stability of a system $\boldsymbol{P}$ implies its finite-gain stability since $\boldsymbol{P}0 = 0$.

2.2 Closed-Loop Stability

Consider a standard closed-loop system $\boldsymbol{P} \,\#\, \boldsymbol{C}$ as illustrated in Fig. 1 with plant $\boldsymbol{P} : \mathcal{L}_{2e}^m \to \mathcal{L}_{2e}^p$ and controller $\boldsymbol{C} : \mathcal{L}_{2e}^p \to \mathcal{L}_{2e}^m$. In the following, the superscripts may be omitted when the dimensions are clear from the context. The graph of $\boldsymbol{P}$ and the inverse graph of $\boldsymbol{C}$ are defined, respectively, as

$$\mathcal{G}_{\boldsymbol{P}} = \begin{bmatrix} \boldsymbol{I} \\ \boldsymbol{P} \end{bmatrix} \mathcal{L}_{2e} \ \text{ and } \ \mathcal{G}'_{\boldsymbol{C}} = \begin{bmatrix} \boldsymbol{C} \\ \boldsymbol{I} \end{bmatrix} \mathcal{L}_{2e}.$$

The corresponding $\mathcal{L}_2$ graphs of $\boldsymbol{P}$ and $\boldsymbol{C}$ are defined as

$$\mathcal{G}_{\boldsymbol{P}}^2 := \mathcal{G}_{\boldsymbol{P}} \cap \mathcal{L}_2 \ \text{ and } \ \mathcal{G}_{\boldsymbol{C}}'^2 := \mathcal{G}'_{\boldsymbol{C}} \cap \mathcal{L}_2,$$

respectively, both of which are assumed to be closed throughout this paper. When P and C are FDLTI, $\mathcal{G}_P^2$ and $\mathcal{G}_C'^2$ are closed subspaces in $\mathcal{L}_2$.

It can be seen in (Willems, 1971; Vidyasagar, 1993; Khong, Cantoni, & Manton, 2013) that various versions of feedback well-posedness may be stipulated based on different signal spaces and causality requirements. In this study, we adopt the well-posedness definition from (Willems, 1971; Vidyasagar, 1993).

Definition 4 *The closed-loop system $\boldsymbol{P} \,\#\, \boldsymbol{C}$ in Fig. 1 is said to be well-posed if*

$$\boldsymbol{F}_{\boldsymbol{P},\boldsymbol{C}} = \begin{bmatrix} u_1 \\ y_2 \end{bmatrix} \mapsto \begin{bmatrix} d_1 \\ d_2 \end{bmatrix} = \begin{bmatrix} \boldsymbol{I} & -\boldsymbol{C} \\ -\boldsymbol{P} & \boldsymbol{I} \end{bmatrix}$$

is causally invertible on $\mathcal{L}_{2e}^{m+p}$.

Throughout the paper, well-posedness will always be assumed for the nominal as well as for all perturbed closed-loop systems. Correspondingly, closed-loop stability is defined as follows.

Definition 5 *A well-posed closed-loop system $\boldsymbol{P} \,\#\, \boldsymbol{C}$ is (finite-gain) stable if $\boldsymbol{F}_{\boldsymbol{P},\boldsymbol{C}}^{-1}$ is finite-gain stable.*

When $\boldsymbol{P} \,\#\, \boldsymbol{C}$ is well-posed, a pair of parallel projection operators (Doyle, Georgiou, & Smith, 1993; Georgiou & Smith, 1997), $\boldsymbol{\Pi}_{\mathcal{G}_{\boldsymbol{P}} /\!/ \mathcal{G}'_{\boldsymbol{C}}}$ along $\mathcal{G}'_{\boldsymbol{C}}$ onto $\mathcal{G}_{\boldsymbol{P}}$ and $\boldsymbol{\Pi}_{\mathcal{G}'_{\boldsymbol{C}} /\!/ \mathcal{G}_{\boldsymbol{P}}}$ along $\mathcal{G}_{\boldsymbol{P}}$ onto $\mathcal{G}'_{\boldsymbol{C}}$, can be defined respectively as

$$\boldsymbol{\Pi}_{\mathcal{G}_{\boldsymbol{P}} /\!/ \mathcal{G}'_{\boldsymbol{C}}} = \begin{bmatrix} d_1 \\ d_2 \end{bmatrix} \mapsto \begin{bmatrix} u_1 \\ y_1 \end{bmatrix} = \boldsymbol{F}_{\boldsymbol{P},\boldsymbol{C}}^{-1} + \begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & -\boldsymbol{I} \end{bmatrix},$$

$$\boldsymbol{\Pi}_{\mathcal{G}'_{\boldsymbol{C}} /\!/ \mathcal{G}_{\boldsymbol{P}}} = \begin{bmatrix} d_1 \\ d_2 \end{bmatrix} \mapsto \begin{bmatrix} u_2 \\ y_2 \end{bmatrix} = \boldsymbol{F}_{\boldsymbol{P},\boldsymbol{C}}^{-1} + \begin{bmatrix} -\boldsymbol{I} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix}.$$

It follows that every $w \in \mathcal{L}_{2e}^{m+p}$ has a unique decomposition $w = u + v$ with $u = \boldsymbol{\Pi}_{\mathcal{G}_{\boldsymbol{P}} /\!/ \mathcal{G}'_{\boldsymbol{C}}} w \in \mathcal{G}_{\boldsymbol{P}}$ and $v = \boldsymbol{\Pi}_{\mathcal{G}'_{\boldsymbol{C}} /\!/ \mathcal{G}_{\boldsymbol{P}}} w \in \mathcal{G}'_{\boldsymbol{C}}$. It is shown in (Doyle et al., 1993) that a well-posed closed-loop system $\boldsymbol{P} \,\#\, \boldsymbol{C}$ is finite-gain stable if and only if $\boldsymbol{\Pi}_{\mathcal{G}_{\boldsymbol{P}} /\!/ \mathcal{G}'_{\boldsymbol{C}}}$ or $\boldsymbol{\Pi}_{\mathcal{G}'_{\boldsymbol{C}} /\!/ \mathcal{G}_{\boldsymbol{P}}}$ is finite-gain stable. Consequently, for a finite-gain stable closed-loop system $\boldsymbol{P} \,\#\, \boldsymbol{C}$, we can define its stability margin as $\|\boldsymbol{\Pi}_{\mathcal{G}_{\boldsymbol{P}} /\!/ \mathcal{G}'_{\boldsymbol{C}}}\|^{-1}$. It is shown in (Doyle et al., 1993) that if either $\boldsymbol{P}$ or $\boldsymbol{C}$ is linear, then $\|\boldsymbol{\Pi}_{\mathcal{G}_{\boldsymbol{P}} /\!/ \mathcal{G}'_{\boldsymbol{C}}}\| = \|\boldsymbol{\Pi}_{\mathcal{G}'_{\boldsymbol{C}} /\!/ \mathcal{G}_{\boldsymbol{P}}}\|$. In particular, when $P \,\#\, C$ is FDLTI, the parallel projection operators reduce to the Gang of Four transfer matrix (Åström & Murray, 2008), i.e.,

$$\boldsymbol{\Pi}_{\mathcal{G}_P /\!/ \mathcal{G}'_C} = \begin{bmatrix} I \\ P \end{bmatrix} (I - CP)^{-1} \begin{bmatrix} I & -C \end{bmatrix} = P \,\#\, C.$$

Henceforth, for an FDLTI closed-loop system, $P \,\#\, C$ denotes both the closed-loop system itself and its Gang of Four transfer matrix.

2.3 Gap Metric and Robust Stability

A well-known tool for characterizing the model uncertainty in a system is the "gap" (or "aperture") and its variants (Zames & El-sakkary, 1980; Georgiou, 1988; Georgiou & Smith, 1990; Qiu & Davison, 1992a; Schumacher, 1992; Qiu & Davison, 1992b; Vinnicombe, 1993). In what follows, we review some concepts related to the gap metric.

Let $\mathcal{X}$ and $\mathcal{Y}$ be two closed subspaces of a Hilbert space $\mathcal{L}$, and let $\Pi_{\mathcal{X}}$ and $\Pi_{\mathcal{Y}}$ be the orthogonal projections onto $\mathcal{X}$ and $\mathcal{Y}$, respectively. The gap metric between the two subspaces is defined as (Kato, 1966, Chapter 2)

$$\gamma(\mathcal{X},\mathcal{Y}) := \|\Pi_{\mathcal{X}} - \Pi_{\mathcal{Y}}\|. \tag{2}$$

The gap between FDLTI systems P_1 and P_2 is defined as the gap between their respective $\mathcal{L}_2$ graphs, i.e.,

$$\delta(P_1,P_2) := \gamma(\mathcal{G}^2_{P_1},\mathcal{G}^2_{P_2}).$$

Given an FDLTI system $P \in \mathcal{P}$, denote the gap ball with center P and radius $r \in [0,1)$ by

$$\mathcal{B}(P,r) := \left\{\tilde{P} \in \mathcal{P} : \ \delta(P,\tilde{P}) \le r\right\}. \tag{3}$$

The following robust stability result, with the stability condition given in terms of an "arcsine" inequality, was obtained in (Qiu & Davison, 1992a). We state it in the following lemma.

Lemma 2 *Let $P \,\#\, C \in \mathcal{RH}_\infty$ and $r_p, r_c \in [0,1)$. The feedback system $\tilde{P} \,\#\, \tilde{C}$ is stable for all $\tilde{P} \in \mathcal{B}(P,r_p)$ and $\tilde{C} \in \mathcal{B}(C,r_c)$ if and only if*

$$\arcsin r_p + \arcsin r_c < \arcsin \|P \,\#\, C\|^{-1}_\infty.$$

3 Main Results: Networked Robust Stability

In this section, we present our main result of the study, which is a necessary and sufficient robust stability condition for a two-port NCS subject to simultaneous uncertainties. We start with some concepts related to two-port networks, and then investigate the stability of such an NCS.

3.1 Uncertain Two-Port Networks

The use of two-port networks as a model of communication channels is adopted from (Gu & Qiu, 2011; Zhao, Qiu, & Gu, 2020). The network $\boldsymbol{T}$ in Fig. 3 has two external ports, with one port composed of v, w and the other of u, y, and is hence called a two-port network. Define its transmission matrix $\boldsymbol{T}$ as

$$\boldsymbol{T} = \begin{bmatrix} \boldsymbol{T}_{11} & \boldsymbol{T}_{12} \\ \boldsymbol{T}_{21} & \boldsymbol{T}_{22} \end{bmatrix} = \begin{bmatrix} u \\ y \end{bmatrix} \mapsto \begin{bmatrix} v \\ w \end{bmatrix}. \tag{4}$$

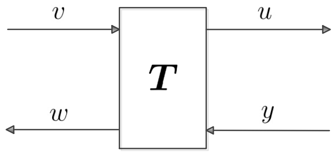

Fig. 3. Single two-port network.

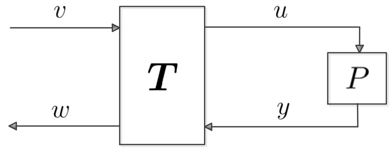

Fig. 4. One-stage two-port connection.

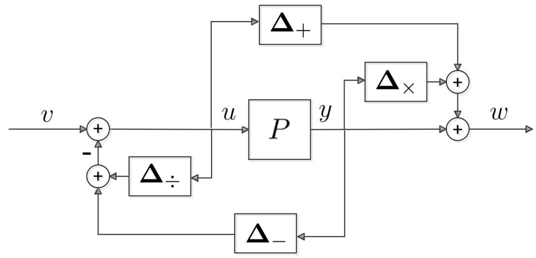

Fig. 5. Plant with the uncertainty quartet.

Henceforth, $\boldsymbol{T}$ stands for both the two-port network itself and its transmission representation for notational simplicity. We model a bidirectional communication channel subject to nonlinear distortions and interferences by a two-port network with its transmission matrix $\boldsymbol{T} = \boldsymbol{I} + \boldsymbol{\Delta}$, where $\boldsymbol{\Delta} = \begin{bmatrix} \boldsymbol{\Delta}_\div & \boldsymbol{\Delta}_- \\ \boldsymbol{\Delta}_+ & \boldsymbol{\Delta}_\times \end{bmatrix}$ is finite-gain stable with $\|\boldsymbol{\Delta}\| < 1$. We also assume that $\boldsymbol{\Delta}$ is strongly causal as introduced in Definition 1. In this case, $\boldsymbol{I} \,\#\, (-\boldsymbol{\Delta})$ is well-posed, and it follows from the nonlinear small-gain theorem (Desoer & Vidyasagar, 1975, Chapter 3) that $\boldsymbol{T}^{-1}$ is causal and stable. The four-block operator matrix $\boldsymbol{\Delta}$ is called the (nonlinear) uncertainty quartet (Zhao, Chen, Khong, & Qiu, 2018).

Fig. 4 describes a two-port network $\boldsymbol{T} = \boldsymbol{I} + \boldsymbol{\Delta}$ connected to an FDLTI system P. One way to analyze how the uncertainties influence the nominal system is via the transformation of the diagram into Fig. 5. It follows from the strong causality of $\boldsymbol{\Delta}_-$ and $\boldsymbol{\Delta}_\div$ that the feedback loop in Fig. 5 is well-posed. As a result, the perturbed system $v \mapsto w$ is well defined and causal.

3.2 Graph Analysis on Cascaded Two-Port NCSs

In order to acquire a better understanding of a two-port NCS, we establish in the following the connection between a two-port NCS and its equivalent closed-loop

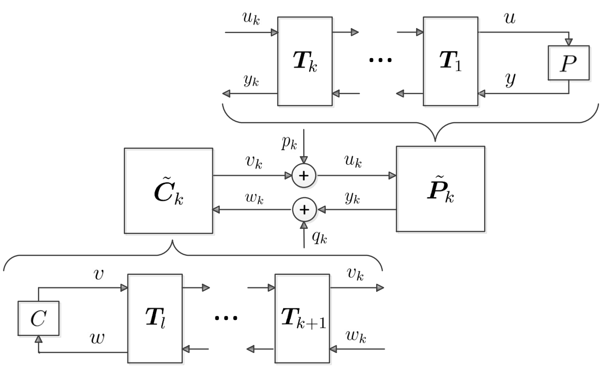

Fig. 6. Perturbed plant and controller representations.

systems by investigating the graphs of systems. As in Fig. 6, the k-th stage two-port network, which admits a strongly causal and incrementally stable nonlinear uncertainty $\boldsymbol{\Delta}_k$, is represented by the transmission matrix $\boldsymbol{T}_k = \boldsymbol{I} + \boldsymbol{\Delta}_k$, $k = 1, 2, \ldots, l$. We can associate the first k stages of the cascaded two-port networks with plant $P \in \mathcal{P}^{p\times m}$, and the remaining $l-k$ stages with the controller $C \in \mathcal{P}^{m\times p}$. Then the diagram in Fig. 2 is equivalently transformed into that in Fig. 6 to form a standard closed-loop system $\tilde{\boldsymbol{P}}_k \# \tilde{\boldsymbol{C}}_k$.

With similar analysis in (Zhao, Qiu, & Gu, 2020, Section III.B), we view P together with $\{\boldsymbol{T}_j\}_{j=1}^k$ as a perturbed plant $\tilde{\boldsymbol{P}}_k = u_k \mapsto y_k$ with uncertainties $\{\boldsymbol{\Delta}_j\}_{j=1}^k$, and thereby obtain the graph of $\tilde{\boldsymbol{P}}_k$ as

$$\mathcal{G}_{\tilde{\boldsymbol{P}}_k} = (\boldsymbol{I} + \boldsymbol{\Delta}_k)\cdots(\boldsymbol{I} + \boldsymbol{\Delta}_1)\mathcal{G}_P. \tag{5}$$

Similarly, we view C together with $\{\boldsymbol{T}_j\}_{j=k+1}^l$ as a perturbed controller $\tilde{\boldsymbol{C}}_k = w_k \mapsto v_k$ with uncertainties $\{\boldsymbol{\Delta}_j\}_{j=k+1}^l$, and obtain the inverse graph of $\tilde{\boldsymbol{C}}_k$ as

$$\mathcal{G}'_{\tilde{\boldsymbol{C}}_k} = (\boldsymbol{I} + \boldsymbol{\Delta}_{k+1})^{-1}\cdots(\boldsymbol{I} + \boldsymbol{\Delta}_l)^{-1}\mathcal{G}'_C. \tag{6}$$

Similarly to the analysis for Fig. 5 in Section 3.1, we obtain from the strong causality of $\boldsymbol{\Delta}_k$, $k = 1, 2, \ldots, l$, that $\tilde{\boldsymbol{P}}_k$ and $\tilde{\boldsymbol{C}}_k$ are well defined and causal.

3.3 Main Theorem: Robust Stability Condition

As shown in Fig. 6, we denote the k-th input pair of the NCS as $I_k := [p_k^T\ q_k^T]^T$, the k-th output pair as $O_k := [u_k^T\ w_k^T]^T$, $k = 1, 2, \ldots, l$, and the vector collecting all outputs as $O := [O_1^T\ O_2^T\ \cdots\ O_l^T]^T$. By the well-posedness assumption, the map from $I_k \in \mathcal{L}_{2e}$ to $O \in \mathcal{L}_{2e}$ is well defined.

Definition 6 *The two-port NCS in Fig. 6 is said to be stable if $I_k \mapsto O$ is finite-gain stable for every $k = 1, 2, \ldots, l$.*

The following proposition further simplifies the stability condition of an NCS.

Proposition 1 *The two-port NCS is finite-gain stable if and only if the closed-loop system $\tilde{\boldsymbol{P}}_k \# \tilde{\boldsymbol{C}}_k$ is finite-gain stable for every $k = 1, 2, ..., l$.*

Given the definition of stability of NCSs, we are ready to present the main robust stability theorem involving simultaneous uncertainties in the two-port NCS. Specifically, the communication channels are subject to nonlinear perturbations, and the plant and controller are subject to model uncertainties characterized by the gap metric. The proof of the theorem is provided in Section 4.

Theorem 1 *Let $P \# C \in \mathcal{RH}_\infty$, P or C be strongly causal, and $r_p, r_c, r_k \in [0, 1)$. The NCS in Fig. 2 is finite-gain stable for all $\tilde{P} \in \mathcal{B}(P, r_p)$, $\tilde{C} \in \mathcal{B}(C, r_c)$, and strongly causal and incrementally stable $\boldsymbol{\Delta}_k$ with $\|\boldsymbol{\Delta}_k\| \le r_k$, $k = 1, 2, \ldots, l$, if and only if*

$$\arcsin r_p + \arcsin r_c + \sum_{k=1}^{l} \arcsin r_k < \arcsin \|P \# C\|_\infty^{-1}. \tag{7}$$

From the above theorem, it is clear that $\|P \# C\|_\infty^{-1}$ can be viewed as the robust stability margin of the two-port NCS. The larger the margin, the more robust the two-port NCS. In addition, we observe that the robust stability margin $\|P \# C\|_\infty^{-1}$ is the same as that in a standard closed-loop system in the presence of gap-type uncertainties as in Lemma 2, whereby the optimally robust controller can be derived by the same approach (Georgiou & Smith, 1990). In addition, the controller synthesis is independent of the structure of the communication channel between the plant and controller, such as the number of two-port connections and how the uncertainty bounds are distributed among all the channels.

4 Derivation of the Main Result

This section is dedicated to the derivation of the main result — Theorem 1. Interested readers are referred to (Zhao, Khong, & Qiu, 2020) for the detailed proofs to the lemmas and propositions in what follows.

4.1 Conelike Uncertainty Sets

In order to characterize nonlinear uncertainties in the spirit of the gap metric, we introduce the notion of conelike uncertainty sets as follows. Let $\mathcal{M}$ be a closed set in $\mathcal{L}_2$. Define the conelike neighborhood centered at $\mathcal{M}$ with radius r as

$$\mathcal{S}(\mathcal{M}, r) := \left\{ v \in \mathcal{L}_2 : \inf_{0\neq u\in\mathcal{M}} \frac{\|v-u\|_2}{\|u\|_2} \le r \right\} \cup \{0\}. \tag{8}$$

If $\mathcal{M}$ is the $\mathcal{L}_2$ graph of a certain FDLTI system, the set $\mathcal{S}(\mathcal{M}, r)$ can be viewed as a closed double cone in $\mathcal{L}_2$, which provides some geometric intuition on modelling system uncertainties with such a neighborhood.

Inspired by the standard gap metric result on FDLTI systems in Lemma 2, we have the following result concerning closed-loop systems subject to nonlinear perturbations.

Proposition 2 *Let $P \# C \in \mathcal{RH}_\infty$ and $r_p, r_c \in [0, 1)$. Then the following statements are equivalent.*

(a) $\boldsymbol{F}_{\tilde{\boldsymbol{P}},\tilde{\boldsymbol{C}}}$ *has a bounded inverse on* $\mathcal{R}(\boldsymbol{F}_{\tilde{\boldsymbol{P}},\tilde{\boldsymbol{C}}})$ *for all* $\tilde{\boldsymbol{P}}$ *with* $\mathcal{G}^2_{\tilde{\boldsymbol{P}}} \subset \mathcal{S}(\mathcal{G}^2_P, r_p)$ *and* $\tilde{\boldsymbol{C}}$ *with* $\mathcal{G}'^2_{\tilde{\boldsymbol{C}}} \subset \mathcal{S}(\mathcal{G}'^2_C, r_c)$*;*
(b) $\mathcal{S}(\mathcal{G}^2_P, r_p) \cap \mathcal{S}(\mathcal{G}'^2_C, r_c) = \{0\}$*;*
(c) $\arcsin r_p + \arcsin r_c < \arcsin \|P \# C\|^{-1}_\infty$.

In the above proposition, we present a necessary and sufficient "pre-stability" condition in terms of an "arcsine" inequality, allowing simultaneous nonlinear perturbations on the plant and controller. Under the condition of statement (a), as long as we can further show that $\mathcal{R}(\boldsymbol{F}_{\tilde{\boldsymbol{P}},\tilde{\boldsymbol{C}}}) = \mathcal{L}_2$, then the closed-loop stability of $\tilde{\boldsymbol{P}} \# \tilde{\boldsymbol{C}}$ follows from Lemma 1. It is worth noting that for nonlinear systems, δ-type gaps and γ-type gaps can be used to characterize uncertain systems through their graphs (Georgiou & Smith, 1997; James, Smith, & Vinnicombe, 2005). In contrast, a conelike neighborhood simply gathers all input-output pairs lying within a prespecified angular distance from its center — the $\mathcal{L}_2$ graph of the nominal system.

4.2 Proof of the Main Theorem

We first introduce in the following several useful lemmas.

Lemma 3 *Let $\mathcal{M}_0 \subset \mathcal{L}_2$ be a closed subspace and $\mathcal{M}_j = \mathcal{S}(\mathcal{M}_{j-1}, r_j)$ for $r_j \in [0, 1)$, $j = 1, 2, \ldots, k$, satisfying $\sum_{j=1}^k \arcsin r_j \le \pi/2$. Then it holds that*

$$\mathcal{M}_k \subset \mathcal{S}\left(\mathcal{M}_0, \sin\left(\sum_{j=1}^k \arcsin r_j\right)\right).$$

In what follows, we introduce an important robust stability result related to the directed nonlinear gap (Georgiou & Smith, 1997). For systems $\boldsymbol{P}_1$ and $\boldsymbol{P}_2$, define the directed nonlinear gap from $\boldsymbol{P}_1$ to $\boldsymbol{P}_2$ as

$$\vec{\delta}(\boldsymbol{P}_1, \boldsymbol{P}_2) := \limsup_{T>0} \sup_{v \in \mathcal{G}_{\boldsymbol{P}_2}} \inf_{\substack{u \in \mathcal{G}_{\boldsymbol{P}_1}, \\ \|\boldsymbol{\Gamma}_T u\|_2 \neq 0}} \frac{\|\boldsymbol{\Gamma}_T(u-v)\|_2}{\|\boldsymbol{\Gamma}_T u\|_2}. \quad (9)$$

The following lemma is adapted from (Georgiou & Smith, 1997, Theorem 3).

Lemma 4 *Let nonlinear system $\boldsymbol{P} \# \boldsymbol{C}$ be finite-gain stable. Then $\tilde{\boldsymbol{P}} \# \boldsymbol{C}$ is finite-gain stable for all $\tilde{\boldsymbol{P}}$ with*

$$\vec{\delta}(\boldsymbol{P}, \tilde{\boldsymbol{P}}) < \|\boldsymbol{\Pi}_{\mathcal{G}_{\boldsymbol{P}} /\!/ \mathcal{G}'_{\boldsymbol{C}}}\|^{-1}.$$

Furthermore, we have the following important lemma relating cascaded two-port uncertainty neighborhoods to the directed nonlinear gap. Given an FDLTI closed-loop system $P \# C$ and incrementally stable uncertainty quartets $\boldsymbol{\Delta}_j$, $j = 1, 2, \ldots, l$, we define a family of perturbed plants and controllers $\tilde{\boldsymbol{P}}_k(\tau)$ and $\tilde{\boldsymbol{C}}_k(\nu)$ parameterized by τ and $\nu \in [0, 1]$, respectively, via

$$\mathcal{G}_{\tilde{\boldsymbol{P}}_k(\tau)} = (\boldsymbol{I} + \tau\boldsymbol{\Delta}_k) \cdots (\boldsymbol{I} + \tau\boldsymbol{\Delta}_1)\mathcal{G}_P,$$
$$\mathcal{G}'_{\tilde{\boldsymbol{C}}_k(\nu)} = (\boldsymbol{I} + \nu\boldsymbol{\Delta}_{k+1})^{-1} \cdots (\boldsymbol{I} + \nu\boldsymbol{\Delta}_l)^{-1}\mathcal{G}'_C.$$

Lemma 5 *For all $\epsilon > 0$, there exists $\delta > 0$ such that for all $\tau, \nu \in [0, 1)$, it holds that*

$$\vec{\delta}(\tilde{\boldsymbol{P}}_k(\tau), \tilde{\boldsymbol{P}}_k(\tau + \delta)) < \epsilon \text{ and } \vec{\delta}(\tilde{\boldsymbol{C}}_k(\nu), \tilde{\boldsymbol{C}}_k(\nu + \delta)) < \epsilon.$$

In other words, $\tilde{\boldsymbol{P}}_k(\tau)$ and $\tilde{\boldsymbol{C}}_k(\nu)$, as functions of τ and ν, respectively, are uniformly continuous with respect to the directed gap.

We are ready to develop the proof to Theorem 1, which borrows the idea of using gap-metric homotopy to establish feedback stability from (Rantzer & Megretski, 1997; Cantoni et al., 2012).

PROOF. First we show the necessity using contrapositive arguments. Assume that inequality (7) does not hold. Then it follows from (Zhao, Qiu, & Gu, 2020, Theorem 1) that there exist systems $\tilde{P}$, $\tilde{C} \in \mathcal{P}$ and stable uncertainties $\Delta_k \in \mathcal{RH}_\infty$, $k = 1, 2, \ldots, l$ satisfying that $\tilde{P} \in \mathcal{B}(P, r_p)$, $\tilde{C} \in \mathcal{B}(C, r_c)$, and $\|\Delta_k\|_\infty \le r_k$, so that $P_q \# C_q \notin \mathcal{H}_\infty$ for an integer $q \in [0, l]$. Here, P_q, C_q are determined, respectively, by $\mathcal{G}_{P_q} = (I + \Delta_q) \cdots (I + \Delta_1)\mathcal{G}_{\tilde{P}}$ and $\mathcal{G}'_{C_q} = (I + \Delta_{q+1})^{-1} \cdots (I + \Delta_l)^{-1}\mathcal{G}'_{\tilde{C}}$. Since every strongly causal FDLTI system admits a strictly proper transfer function representation, it follows from the hypothesis that either P or C is strictly proper. Therefore, one can verify that there exists an $\hat{\omega} \neq \infty$ such that $\hat{\omega} \in \arg\max_{\omega \in \mathbb{R} \cup \infty} \bar{\sigma}(P(j\omega) \# C(j\omega))$. By the proof of necessity of (Zhao, Qiu, & Gu, 2020, Theorem 1) and using the interpolation method in (Vinnicombe, 2000, Lemma 1.14) for $\hat{\omega}$, we can further require that $\Delta_k \in \mathcal{RH}_\infty$, $k = 1, 2, \ldots, l$, are strictly proper transfer matrices, i.e., they are strongly causal. Therefore, by contraposition and Proposition 1, we prove the necessity of the robust stability condition in Theorem 1.

In the rest of this proof, we show the sufficiency of the robust stability condition in three steps.
Step 1: Suppose that we are at the k-th stage of equivalent closed-loop systems as shown in Fig. 6. Let

$$\mathcal{M} = \mathcal{G}_P^2,\ \tilde{\mathcal{M}}_0 = \mathcal{G}_{\tilde{P}}^2,\ \tilde{\mathcal{M}}_j(\tau) = \mathcal{G}^2_{\tilde{\boldsymbol{P}}_j(\tau)},\ j = 1, 2, \ldots, k,$$

where $\mathcal{G}_{\tilde{\boldsymbol{P}}_j(\tau)} = (\boldsymbol{I} + \tau\boldsymbol{\Delta}_j)\cdots(\boldsymbol{I} + \tau\boldsymbol{\Delta}_1)\mathcal{G}_{\tilde{P}}$. Then it follows that

$$\tilde{\mathcal{M}}_0 \subset \mathcal{S}(\mathcal{M}, r_p) \text{ and } \tilde{\mathcal{M}}_j(\tau) = (\boldsymbol{I} + \tau\boldsymbol{\Delta}_j)\tilde{\mathcal{M}}_{j-1}(\tau),$$

with $\|\tau\boldsymbol{\Delta}_j\| \le \tau r_j$. Let $v \in \tilde{\mathcal{M}}_j(\tau) \setminus \{0\}$, then there exists $u_1 \in \tilde{\mathcal{M}}_{j-1}(\tau)$ such that $v = (\boldsymbol{I} + \tau\boldsymbol{\Delta}_j)u_1$. Hence we have

$$\inf_{0 \ne u \in \tilde{\mathcal{M}}_{j-1}(\tau)} \frac{\|v - u\|_2}{\|u\|_2} \le \frac{\|\tau\boldsymbol{\Delta}_j u_1\|_2}{\|u_1\|_2} \le \|\tau\boldsymbol{\Delta}_j\| \le \tau r_j.$$

As a result, $\tilde{\mathcal{M}}_j(\tau) \subset \mathcal{S}(\tilde{\mathcal{M}}_{j-1}(\tau), \tau r_j)$, $j = 1, 2, \ldots, k$. Since $\tilde{\mathcal{M}}_0$ is a closed subspace in $\mathcal{L}_2$, it follows from Lemma 3 and (8) that

$$\tilde{\mathcal{M}}_k(\tau) \subset \mathcal{S}\left(\tilde{\mathcal{M}}_0, \sin\left(\sum_{j=1}^{k} \arcsin \tau r_j\right)\right)$$
$$\subset \mathcal{S}\left(\mathcal{M}, \sin\left(\arcsin r_p + \sum_{j=1}^{k} \arcsin \tau r_j\right)\right).$$

Likewise, for the controller part, let

$$\mathcal{N} = \mathcal{G}_C'^2,\ \tilde{\mathcal{N}}_l = \mathcal{G}_{\tilde{C}}'^2,\ \tilde{\mathcal{N}}_j(\nu) = \mathcal{G}'^2_{\tilde{\boldsymbol{C}}_j(\nu)}, j = k+1, \ldots, l,$$

where $\mathcal{G}'_{\tilde{\boldsymbol{C}}_j(\nu)} = (\boldsymbol{I} + \nu\boldsymbol{\Delta}_{j+1})^{-1}\cdots(\boldsymbol{I} + \nu\boldsymbol{\Delta}_l)^{-1}\mathcal{G}'_{\tilde{C}}$. Then it follows that

$$\tilde{\mathcal{N}}_k(\nu) \subset \mathcal{S}\left(\mathcal{N}, \sin\left(\arcsin r_c + \sum_{j=k+1}^{l} \arcsin \nu r_j\right)\right).$$

Therefore, from the stability condition (7) and Proposition 2, we know $\boldsymbol{F}_{\tilde{\boldsymbol{P}}_k(\tau), \tilde{\boldsymbol{C}}_k(\nu)}$ has a uniformly bounded inverse on $\mathcal{R}(\boldsymbol{F}_{\tilde{\boldsymbol{P}}_k(\tau), \tilde{\boldsymbol{C}}_k(\nu)})$ over all $\tau, \nu \in [0, 1]$, which ensures the existence of a constant $\alpha > 0$ such that

$$\left\|\boldsymbol{\Pi}_{\mathcal{G}_{\tilde{\boldsymbol{P}}_j(\tau)} /\!\!/ \mathcal{G}'_{\tilde{\boldsymbol{C}}_j(\nu)}} x\right\|_2 \le \alpha \|x\|_2, \tag{10}$$

for all $x \in \mathcal{R}(\boldsymbol{F}_{\tilde{\boldsymbol{P}}_k(\tau), \tilde{\boldsymbol{C}}_k(\nu)})$ and $\tau, \nu \in [0, 1]$.

Step 2: Note that $\tilde{\boldsymbol{P}}_k(\tau)$ and $\tilde{\boldsymbol{C}}_k(\nu)$ are uniformly continuous with respect to the directed nonlinear gap. In particular, it follows from Lemma 5 that for $\alpha > 0$ given in (10), there exists $\delta > 0$ such that for all $\tau, \nu \in [0, 1)$,

$$\vec{\delta}(\tilde{\boldsymbol{P}}_k(\tau), \tilde{\boldsymbol{P}}_k(\tau + \delta)) < \frac{1}{\alpha},$$
$$\text{and } \vec{\delta}(\tilde{\boldsymbol{C}}_k(\nu), \tilde{\boldsymbol{C}}_k(\nu + \delta)) < \frac{1}{\alpha}. \tag{11}$$

Step 3: When $\tau = \nu = 0$, it follows from Lemma 2 that $\tilde{\boldsymbol{P}}_k(0) \,\#\, \tilde{\boldsymbol{C}}_k(0) = \tilde{P} \,\#\, \tilde{C}$ is stable. Hence (10) implies that $\left\|\boldsymbol{\Pi}_{\mathcal{G}_{\tilde{\boldsymbol{P}}_j(0)} /\!\!/ \mathcal{G}'_{\tilde{\boldsymbol{C}}_j(0)}}\right\| \le \alpha$. Then combining (11) with Lemma 4, we obtain the finite-gain stability of $\tilde{\boldsymbol{P}}_k(\delta) \,\#\, \tilde{\boldsymbol{C}}_k(0)$. By iteratively using (10), (11) and Lemma 4, we obtain that all the closed-loop systems in the following sequence are finite-gain stable:

$$\tilde{\boldsymbol{P}}_k(\delta) \,\#\, \tilde{\boldsymbol{C}}_k(\delta),\ \tilde{\boldsymbol{P}}_k(2\delta) \,\#\, \tilde{\boldsymbol{C}}_k(\delta),\ \tilde{\boldsymbol{P}}_k(2\delta) \,\#\, \tilde{\boldsymbol{C}}_k(2\delta),$$
$$\ldots, \tilde{\boldsymbol{P}}_k(1) \,\#\, \tilde{\boldsymbol{C}}_k(1 - \delta),\ \tilde{\boldsymbol{P}}_k(1) \,\#\, \tilde{\boldsymbol{C}}_k(1).$$

The finite-gain stability of the two-port NCS then follows by Proposition 1. □

5 Conclusions and Future Work

We investigate the robust stabilization problem of a two-port NCS where the plant and controller are subject to gap-type uncertainties and the cascaded two-port communication channels are subject to nonlinear perturbations. In order to characterize such nonlinear uncertainty, a special conelike neighborhood, which is inspired and motivated by the elegant geometric properties of the gap metric, is proposed and investigated. A necessary and sufficient robust stability condition for the two-port NCS is given in the form of an "arcsine" inequality. The associated robust controller synthesis problem can be settled by solving a special one-block $\mathcal{H}_\infty$ control problem.

We can generalize the current problem setup by modeling communication channels as two-port networks with various types of interconnections, such as cascade, parallel, series, hybrid and so on. In terms of technical developments, we can extend the current model of two-port NCSs in the language of the behaviour approach (Polderman & Willems, 1998) so as to incorporate non-invertible equipments, such as quantizers.

References

Åström, K. J., & Murray, R. M. (2008). *Feedback fundamentals – control & dynamical systems.* Princeton, NJ: Princeton University Press.

Cantoni, M., Jönsson, U. T., & Kao, C.-Y. (2012). Robustness analysis for feedback interconnections of distributed systems via integral quadratic constraints. *IEEE Trans. Automat. Contr.*, *57*(2), 302–317.

Desoer, C. A., & Vidyasagar, M. (1975). *Feedback systems: Input-output properties* (Vol. 55). Philadelphia: SIAM.

Doyle, J. C., Georgiou, T. T., & Smith, M. C. (1993). The parallel projection operators of a nonlinear feedback system. *Syst. Contr. Lett.*, *20*(2), 79 - 85.

Feintuch, A., & Saeks, R. (1982). *System theory: A Hilbert space approach.* London: Academic Press.

Georgiou, T. T. (1988). On the computation of the gap metric. *Syst. Contr. Lett.*, *11*(4), 253–257.

Georgiou, T. T., & Smith, M. C. (1990). Optimal robustness in the gap metric. *IEEE Trans. Automat. Contr.*, *35*(6), 673–686.

Georgiou, T. T., & Smith, M. C. (1993). Graphs, causality, and stabilizability: Linear, shift-invariant systems on $\mathcal{L}_2[0,\infty)$. *Mathematics of Control, Signals and Systems*, *6*(3), 195–223.

Georgiou, T. T., & Smith, M. C. (1997). Robustness analysis of nonlinear feedback systems: An input-output approach. *IEEE Trans. Automat. Contr.*, *42*(9), 1200–1221.

Gu, G., & Qiu, L. (2011). A two-port approach to networked feedback stabilization. *in Proc. 50th IEEE Conf. on Decision and Contr. and European Contr. Conf. (CDC-ECC)*, 2387-2392.

James, M. R., Smith, M. C., & Vinnicombe, G. (2005). Gap metrics, representations, and nonlinear robust stability. *SIAM J. on Contr. Optimization*, *43*(5), 1535–1582.

Kato, T. (1966). *Perturbation theory for linear operators.* New York, NY: Springer.

Khong, S. Z., Cantoni, M., & Manton, J. H. (2013). A gap metric perspective of well-posedness for nonlinear feedback interconnections. *Australian Contr. Conf. (AUCC)*, 224-229.

Kimura, H. (1996). *Chain-scattering approach to $\mathcal{H}_\infty$ control.* New York: Springer Science & Business Media.

Megretski, A., & Rantzer, A. (1997). System analysis via integral quadratic constraints. *IEEE Trans. Automat. Contr.*, *42*(6), 819–830.

Polderman, J. W., & Willems, J. C. (1998). *Introduction to mathematical systems theory: A behavioral approach.* New York, NY: Springer-Verlag.

Qiu, L., & Davison, E. J. (1992a). Feedback stability under simultaneous gap metric uncertainties in plant and controller. *Syst. Contr. Lett.*, *18*(1), 9–22.

Qiu, L., & Davison, E. J. (1992b). Pointwise gap metrics on transfer matrices. *IEEE Trans. Automat. Contr.*, *37*(6), 741–758.

Rantzer, A., & Megretski, A. (1997). *System analysis via integral quadratic constraints: Part II.* Technical Reports LUTFD2/TFRT7559SE, Department of Automatic Control, Lund Institute of Technology.

Schumacher, J. M. (1992). A pointwise criterion for controller robustness. *Syst. Contr. Lett.*, *18*(1), 1–8.

Teel, A. (1996). On graphs, conic relations, and input-output stability of nonlinear feedback systems. *IEEE Trans. Automat. Contr.*, *41*(5), 702–709.

van der Schaft, A. (2017). *$\mathcal{L}_2$-gain and passivity techniques in nonlinear control* (3rd ed.). Switzerland: Springer.

Vidyasagar, M. (1993). *Nonlinear system analysis.* Englewood Cliffs, NJ: Prentice Hall.

Vinnicombe, G. (1993). Frequency domain uncertainty and the graph topology. *IEEE Trans. Automat. Contr.*, *38*(9), 1371–1383.

Vinnicombe, G. (2000). *Uncertainty and feedback: $\mathcal{H}_\infty$ loop-shaping and the ν-gap metric.* Singapore: World Scientific.

Willems, J. C. (1971). *The analysis of feedback systems.* Clinton, Massachusetts: The M.I.T Press.

Zames, G., & El-sakkary, A. K. (1980). Unstable systems and feedback: The gap metric. *in Proc. 16th Allerton Conf.*, 380–385.

Zhang, W., Branicky, M. S., & Phillips, S. M. (2001). Stability of networked control systems. *IEEE Contr. Syst.*, *21*(1), 84–99.

Zhao, D., Chen, C., Khong, S. Z., & Qiu, L. (2018). Robust control against uncertainty quartet: A polynomial approach. In T. Başar (Ed.), *Uncertainty in complex networked systems: In honor of Roberto Tempo* (pp. 149–178). Cham: Springer International Publishing.

Zhao, D., Khong, S. Z., & Qiu, L. (2017). Stabilization of cascaded two-port networked systems against nonlinear perturbations. *in Proc. 56th IEEE Conf. on Decision and Contr. (CDC)*, 1042-1045.

Zhao, D., Khong, S. Z., & Qiu, L. (2020). Stabilization of cascaded two-port networked systems with simultaneous nonlinear uncertainties. *arXiv: 2008.02152*.

Zhao, D., Qiu, L., & Gu, G. (2020). Stabilization of two-port networked systems with simultaneous uncertainties in plant, controller, and communication channels. *IEEE Trans. Automat. Contr.*, *65*(3), 1160-1175.

Zhou, K., & Doyle, J. C. (1998). *Essentials of robust control.* Upper Saddle River, NJ: Prentice Hall.